Avoiding barren plateaus via transferability of smooth solutions in Hamiltonian Variational Ansatz

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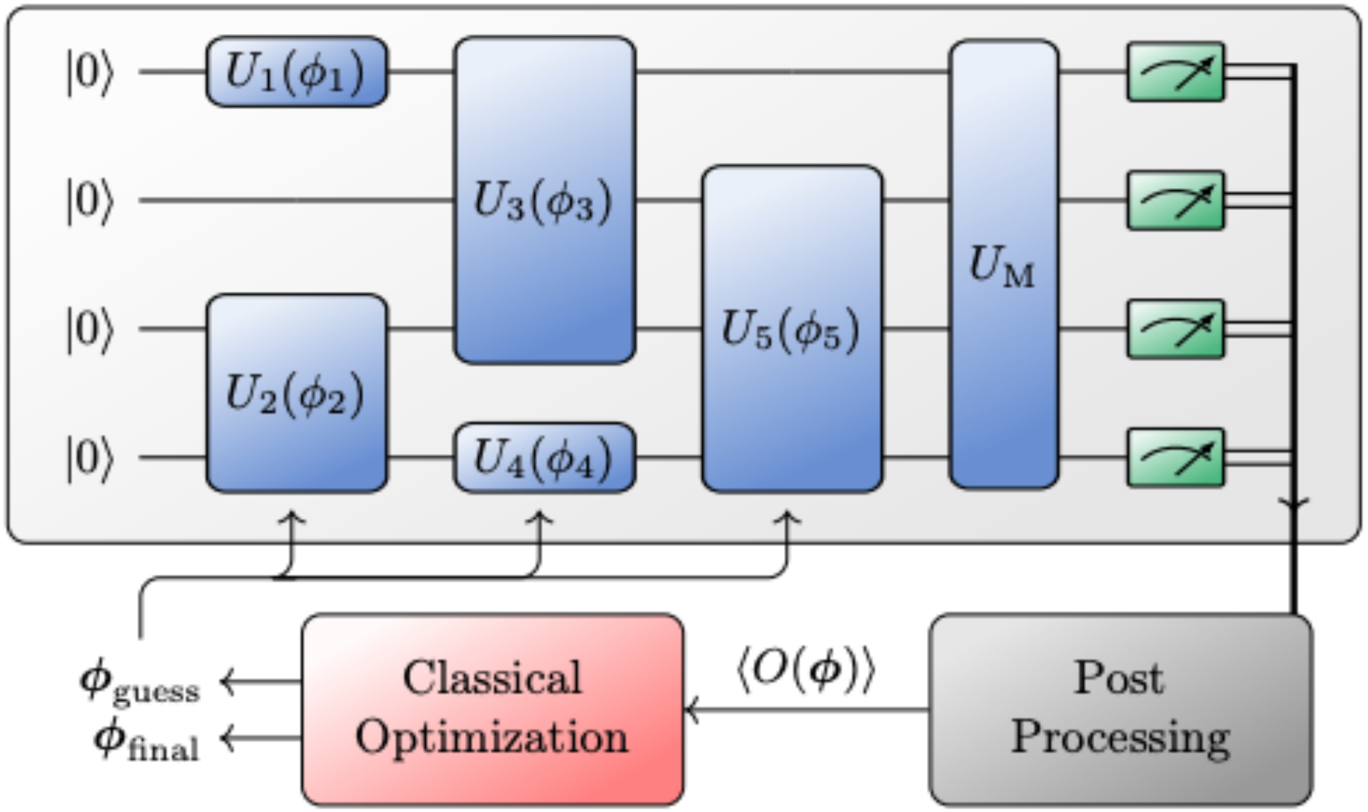
Variational Quantum Algorithms

- Leading **NISQ strategy**

- The problem is encoded in **minimising a cost function**

(e.g. finding Ground state of an Hamiltonian)

(e.g. Hamiltonian expectation value)



VQAs scheme [Bittel et al., PRL (2021)]

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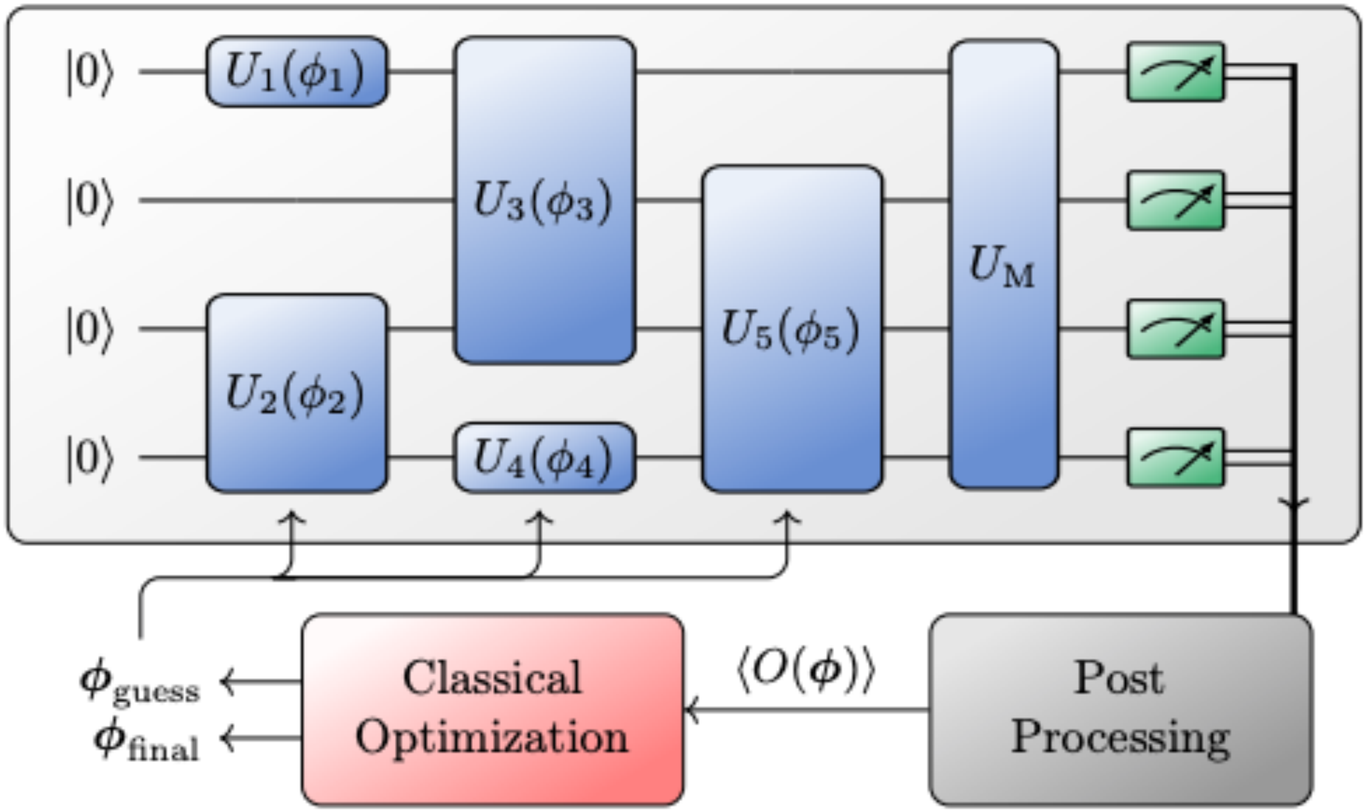

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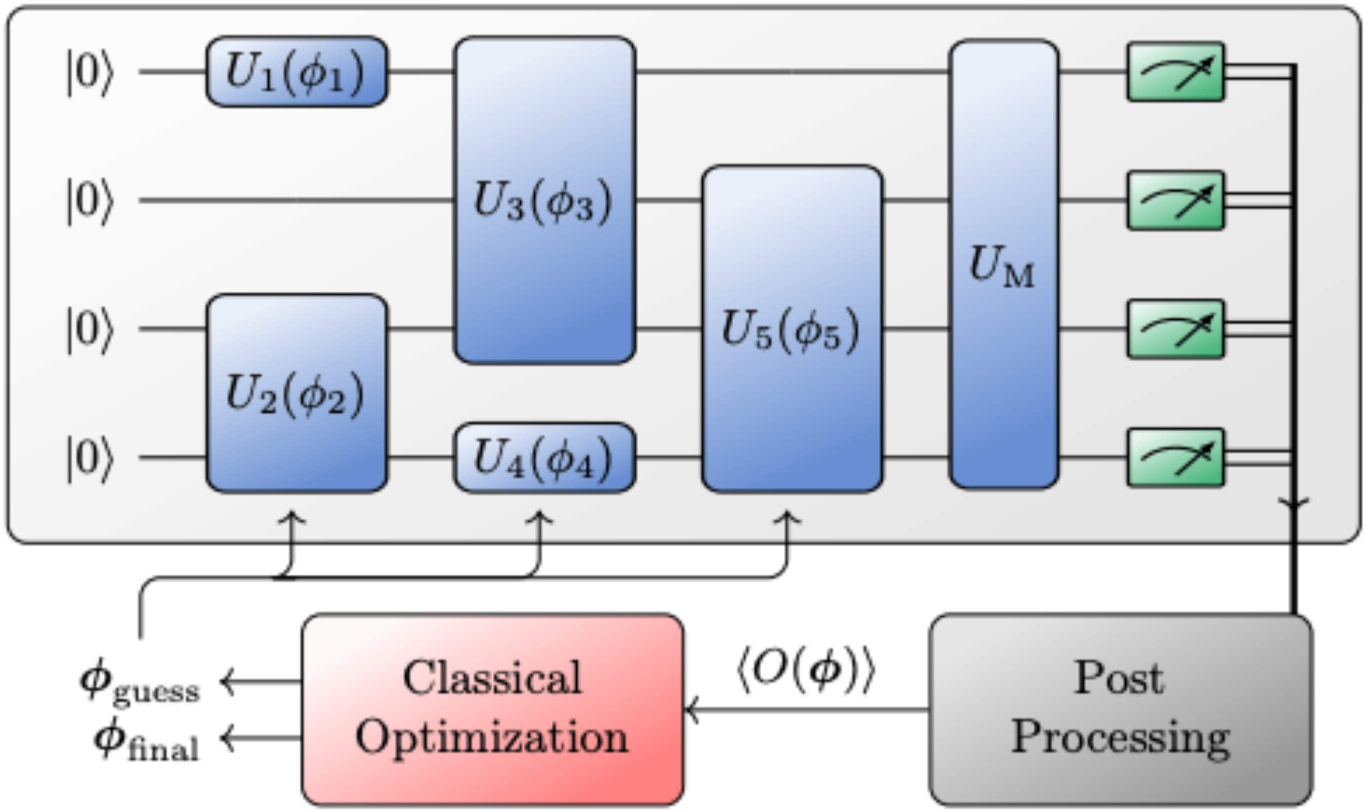

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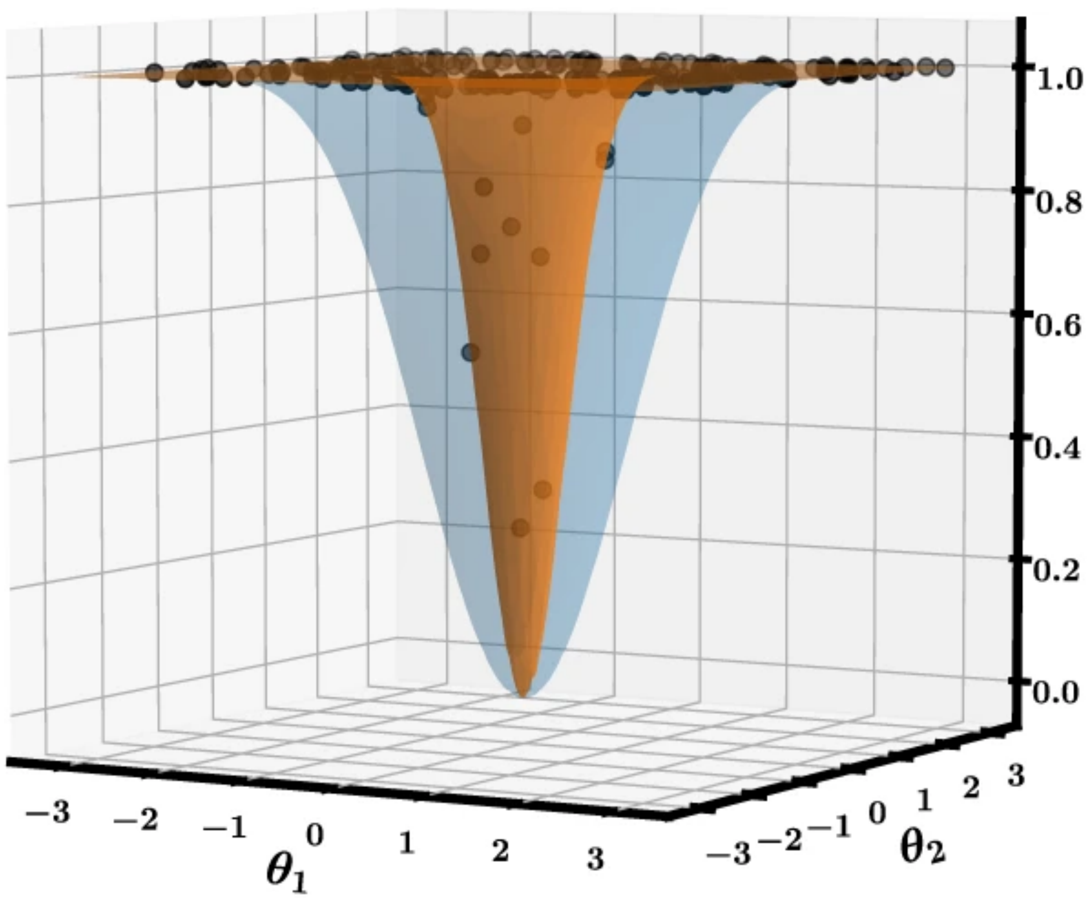
MAIN DIFFICULTIES:

- **Noise**
- **Non-convex** optimization
- Flat landscape (a.k.a. **Barren Plateaus**)

Barren Plateaus

High circuit expressibility → **Exponential vanishing gradients with number of qubits N**
(Barren Plateaus definition)

[McClean et al., Nat. Comm. (2018)]
[Holmes et al., PRX Q. (2022)]



Orange (blue) landscape N = 24 (N = 4) qubits. [Cerezo et al.,Nature(2021)]

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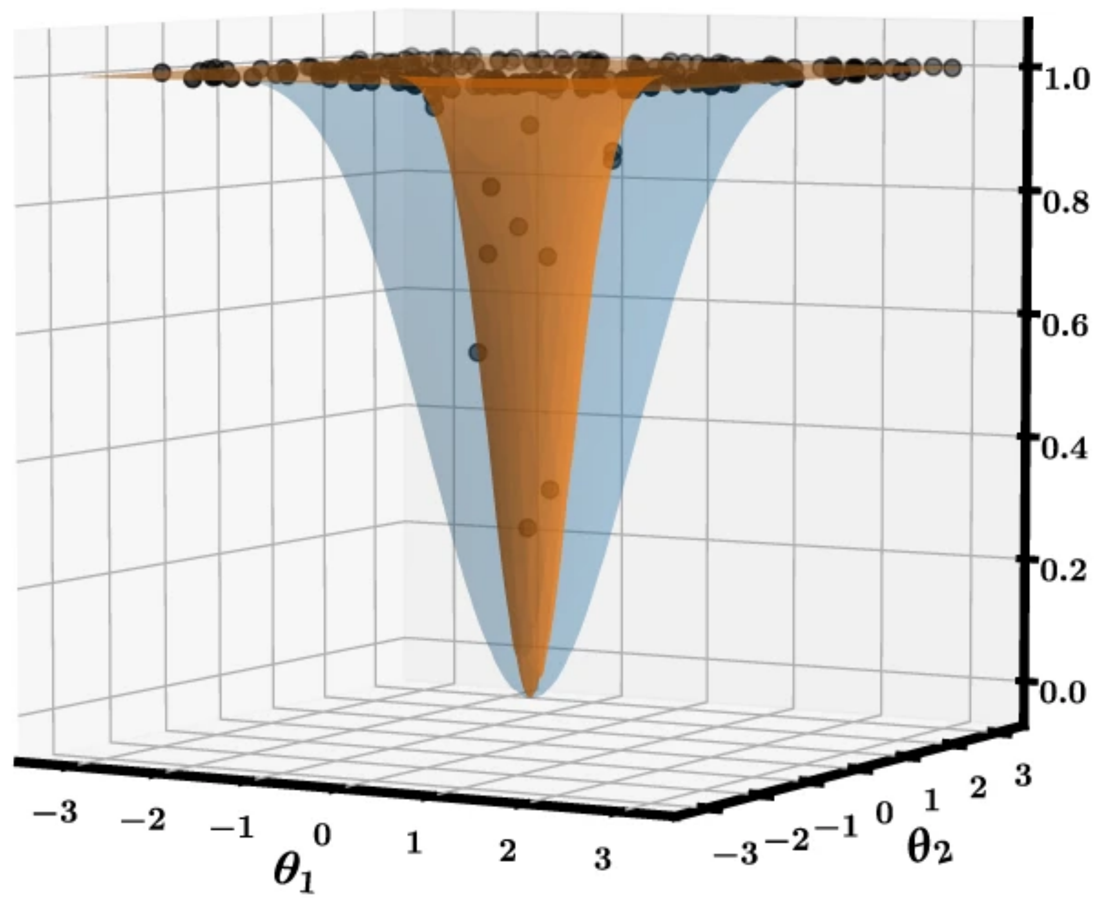

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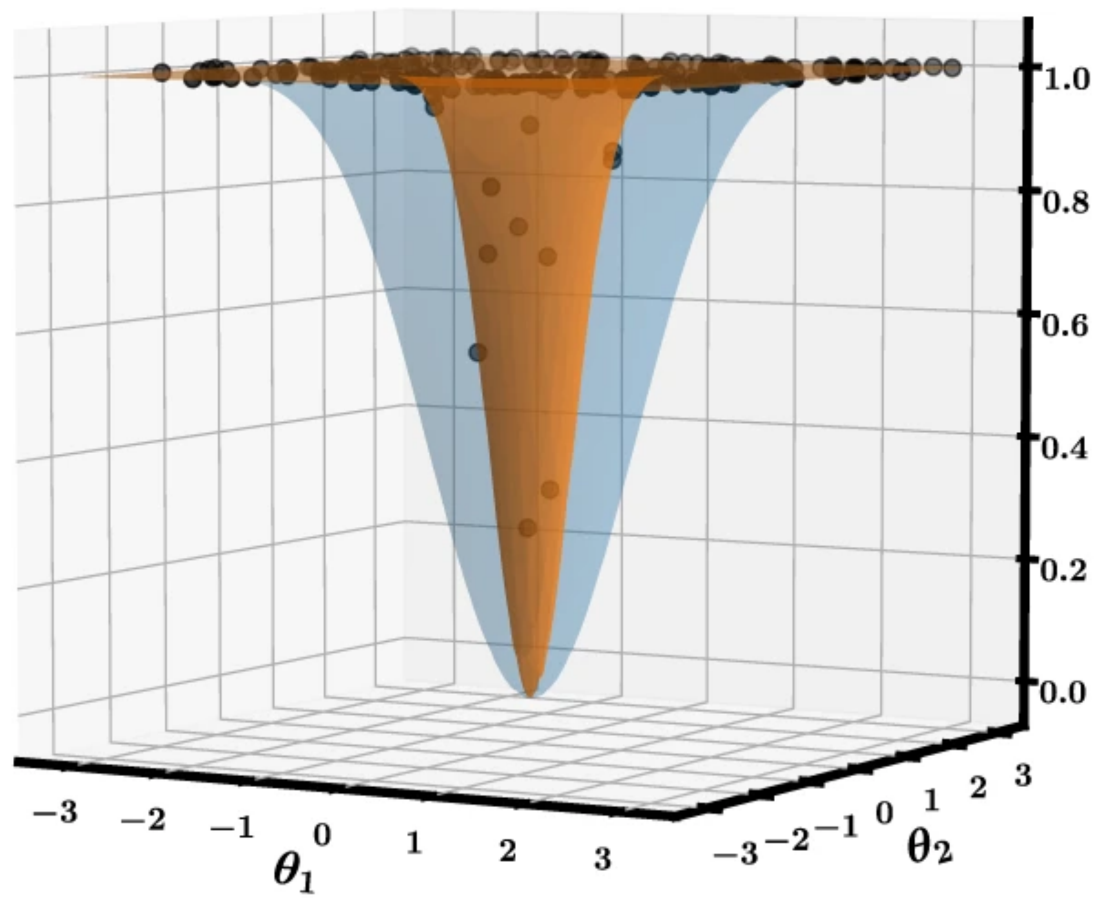

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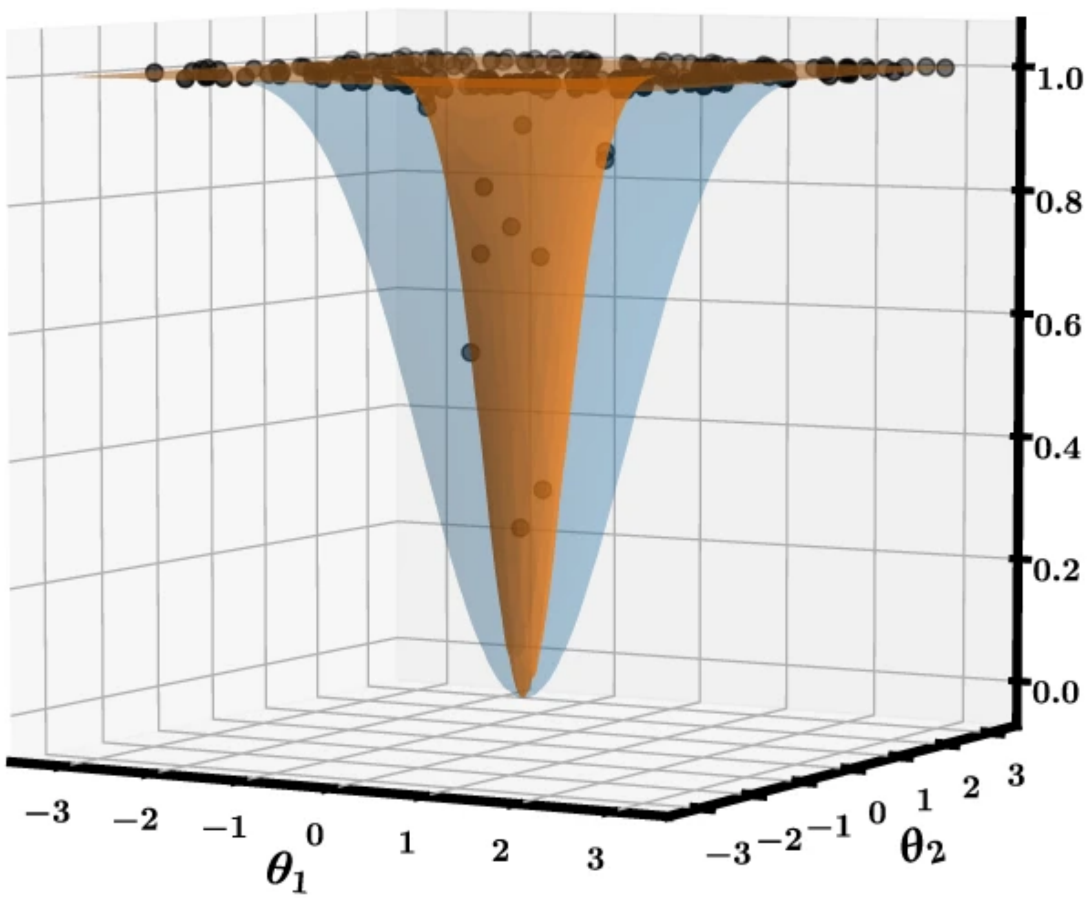
☠️ **Serious problem for VQAs** ☠️



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Problem-inspired Ansatz

$$|\psi(\boldsymbol{\gamma})\rangle = \prod_{m=1}^{P} e^{-i\gamma_{m,M}H_M} \cdots e^{-i\gamma_{m,1}H_1} |\psi_0\rangle$$

The problem **Hamiltonian** is a **linear combination of these generators**

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We analyzed:

$$H_{\mathrm{XYZ}} = \sum_{j=1}^{N}\left(X_jX_{j+1} + \Delta_Y Y_jY_{j+1} + \Delta_Z Z_jZ_{j+1}\right)$$

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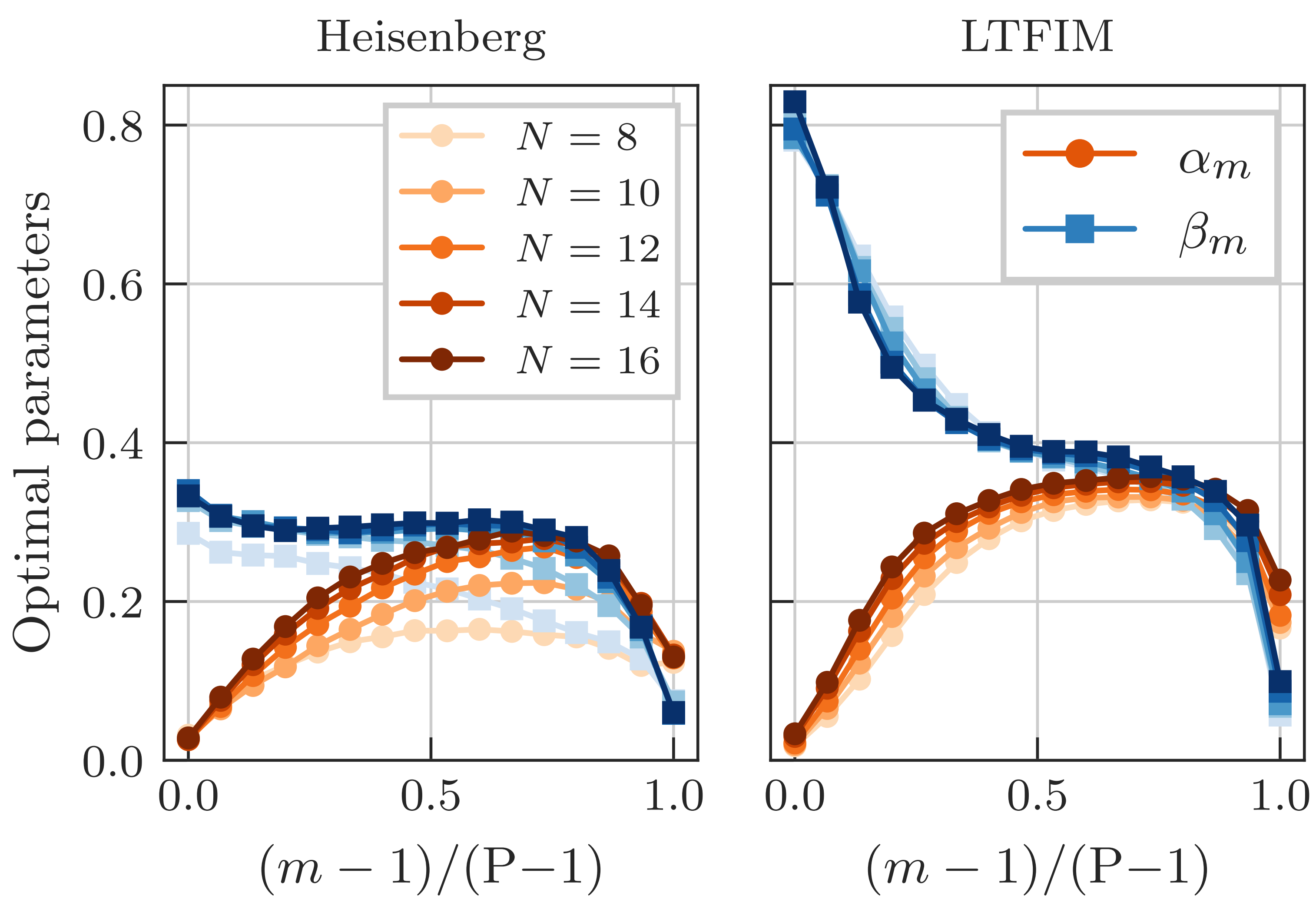
Although symmetry-ansatz, there can be Barren Plateaus

[Larocca et al., ArXiv (2021)]

Pattern in Optimal Parameters

$(\alpha_1, \ldots, \alpha_P, \beta_1, \ldots, \beta_P,)$



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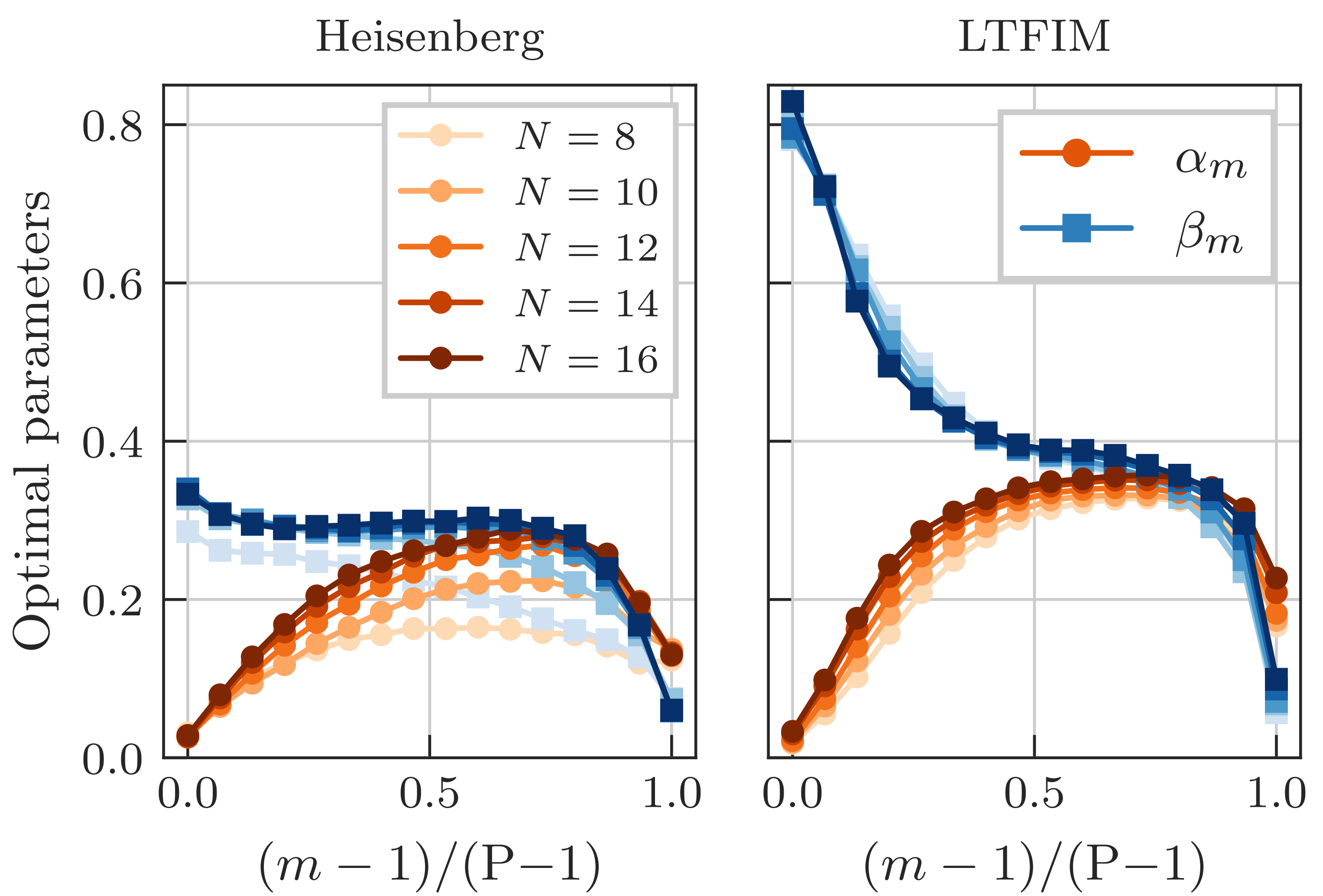

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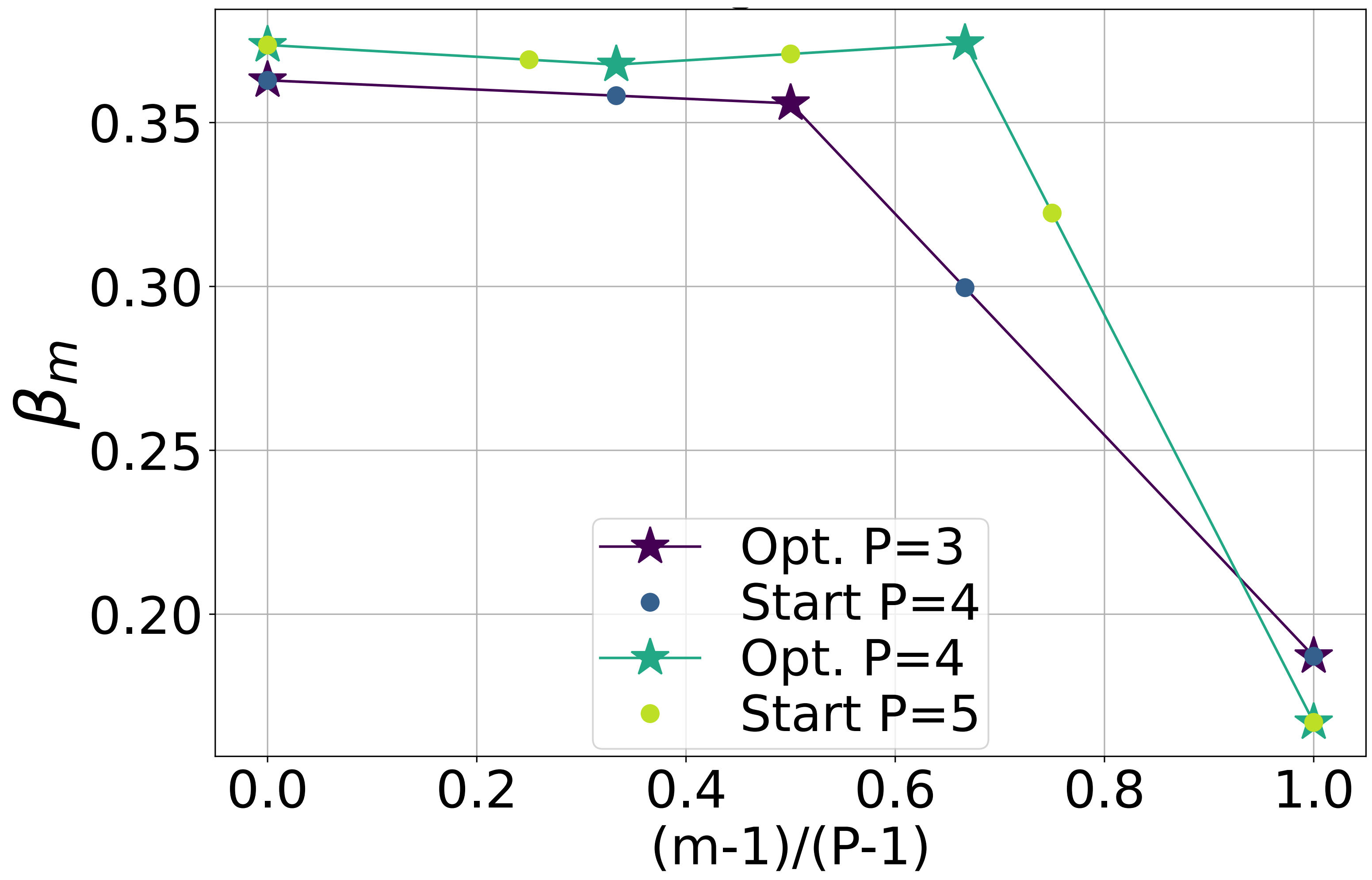
How can we find this pattern?

INTERP strategy

[Zhou et al., PRX. (2018)]



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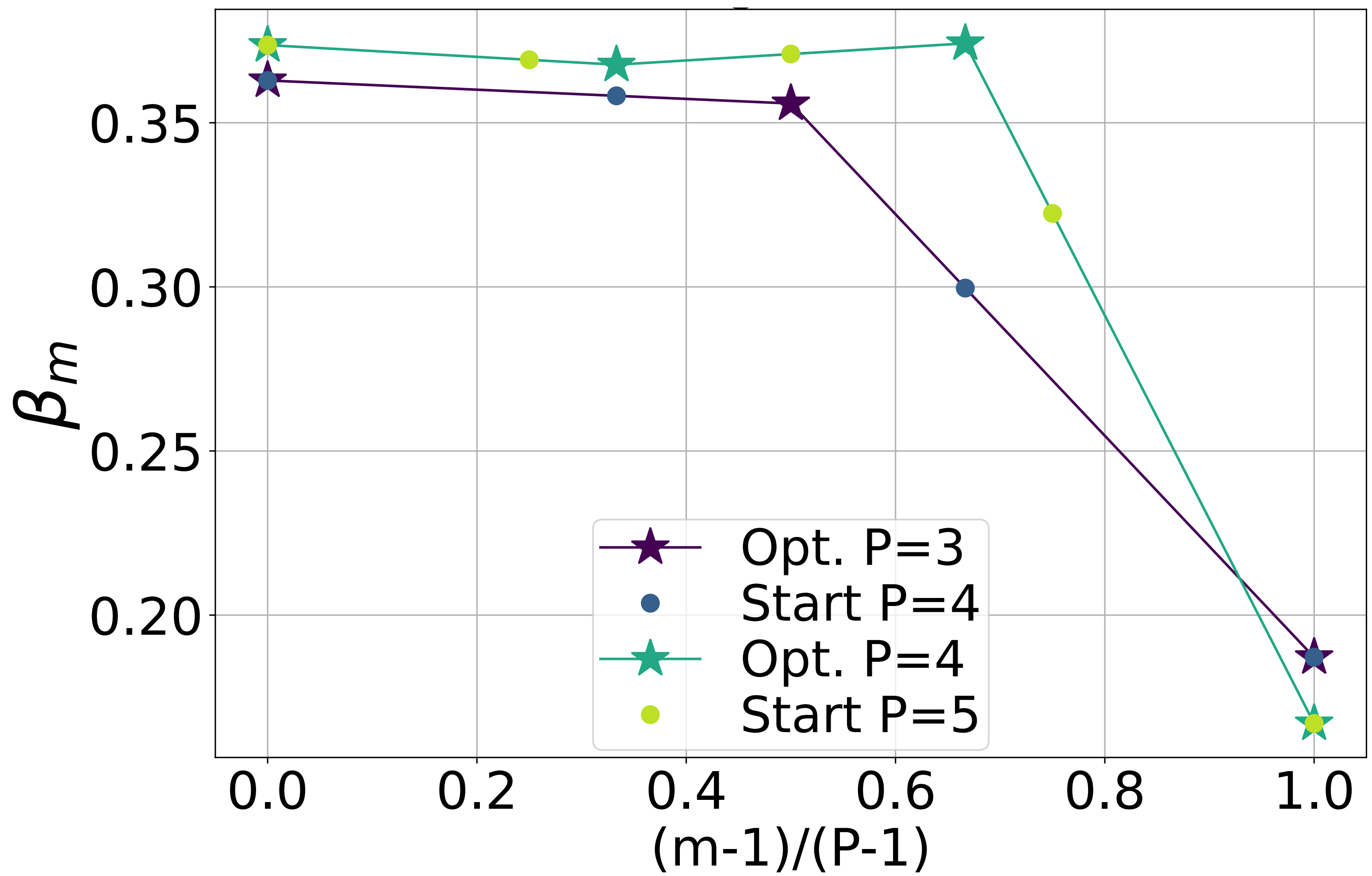
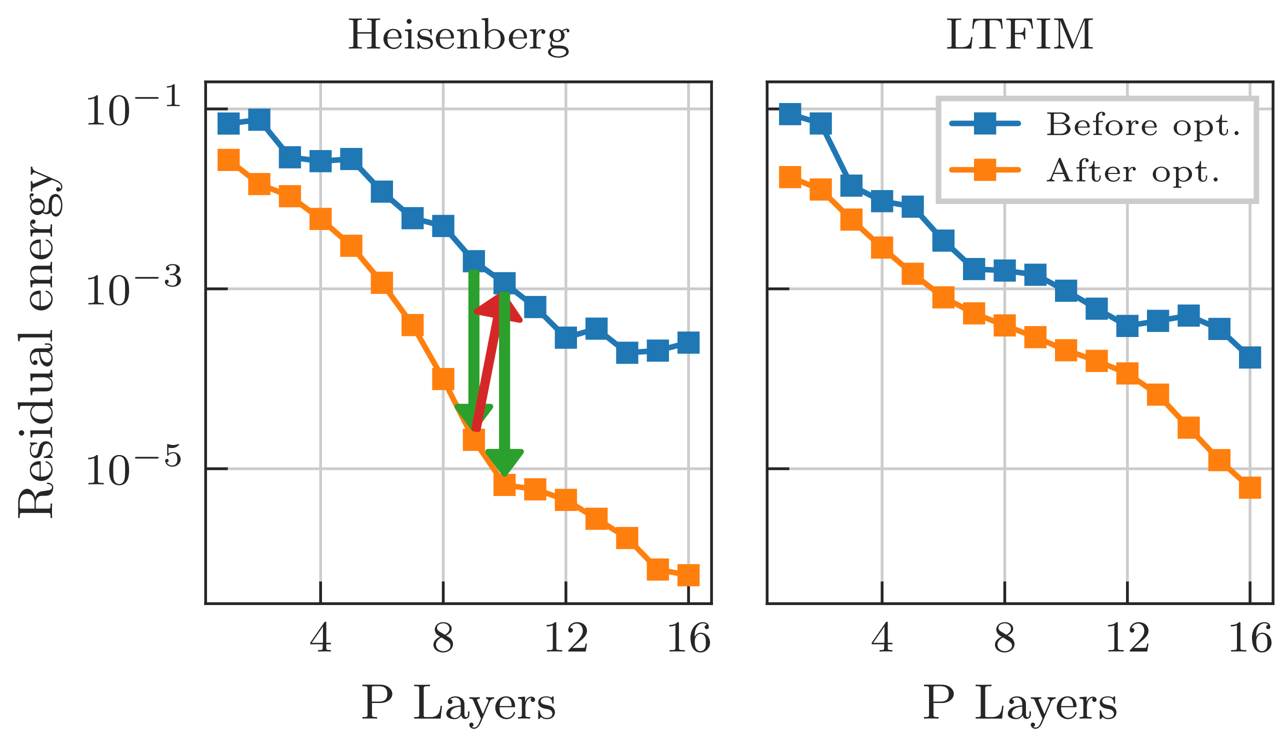
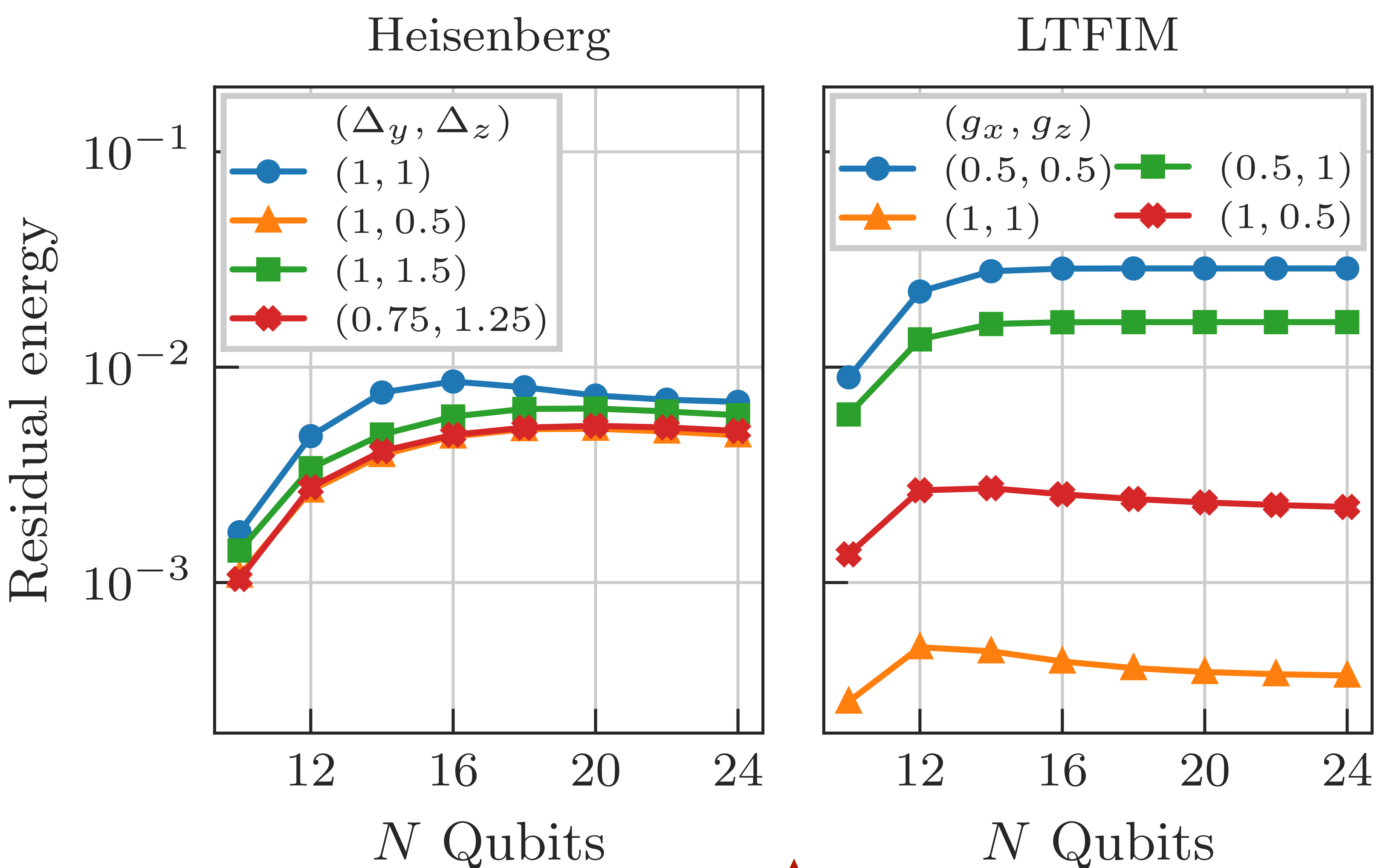


Warm-start for the optimization when you go from P→P+1



Transferability of solutions



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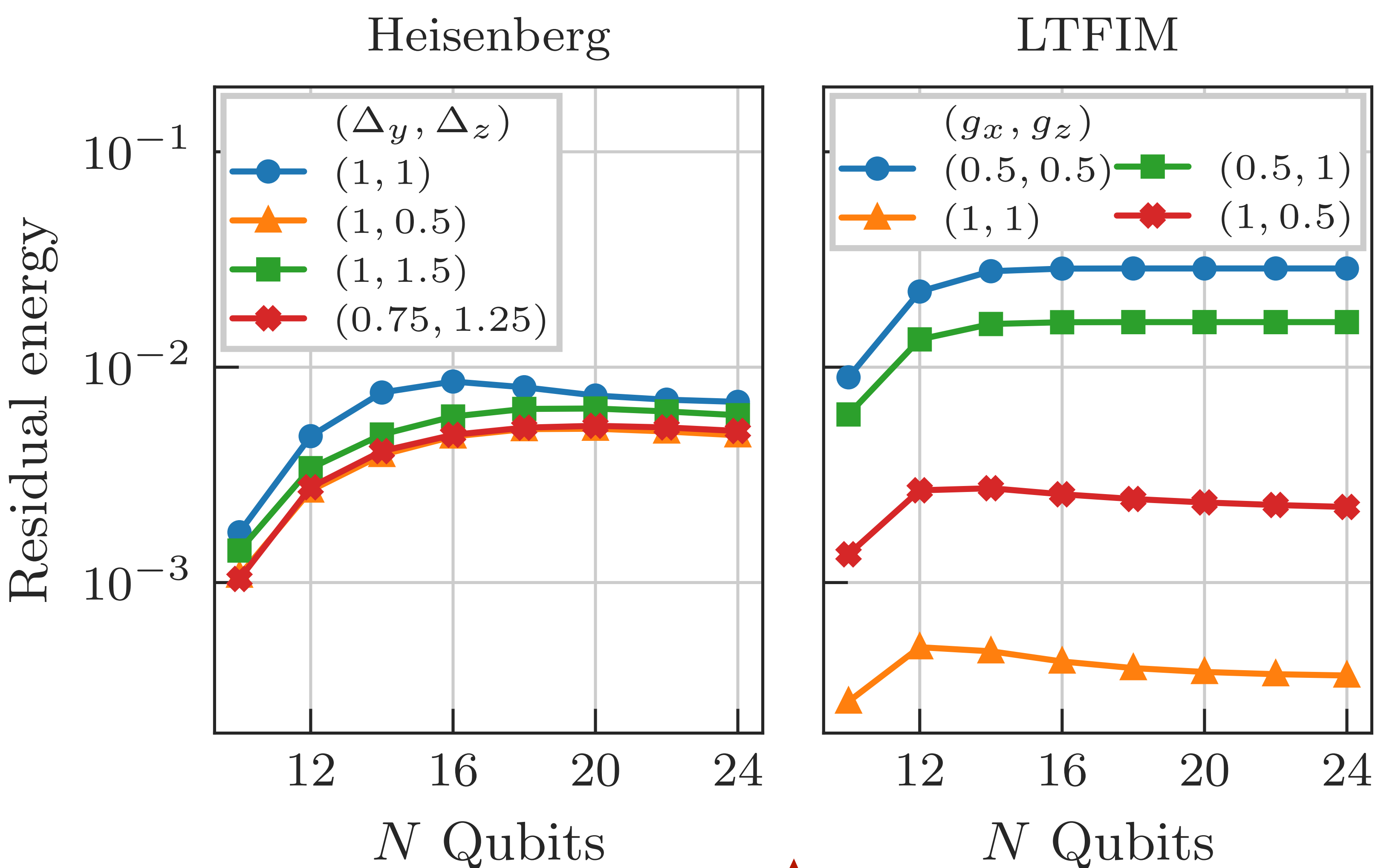

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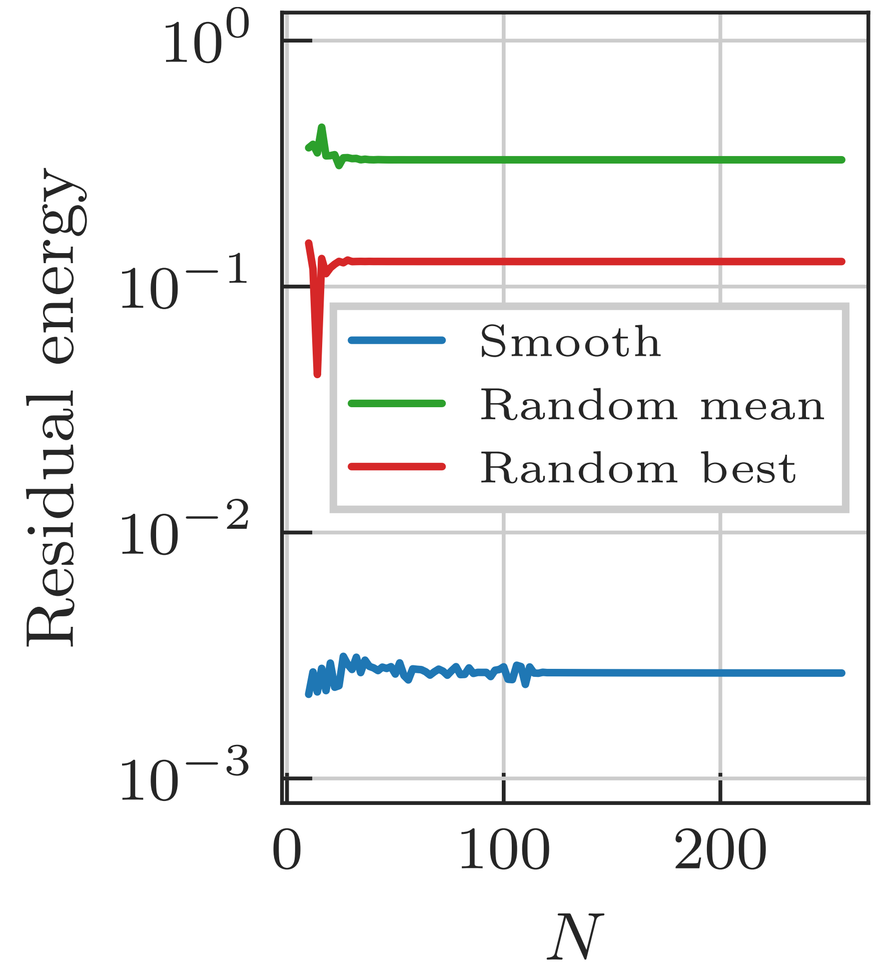
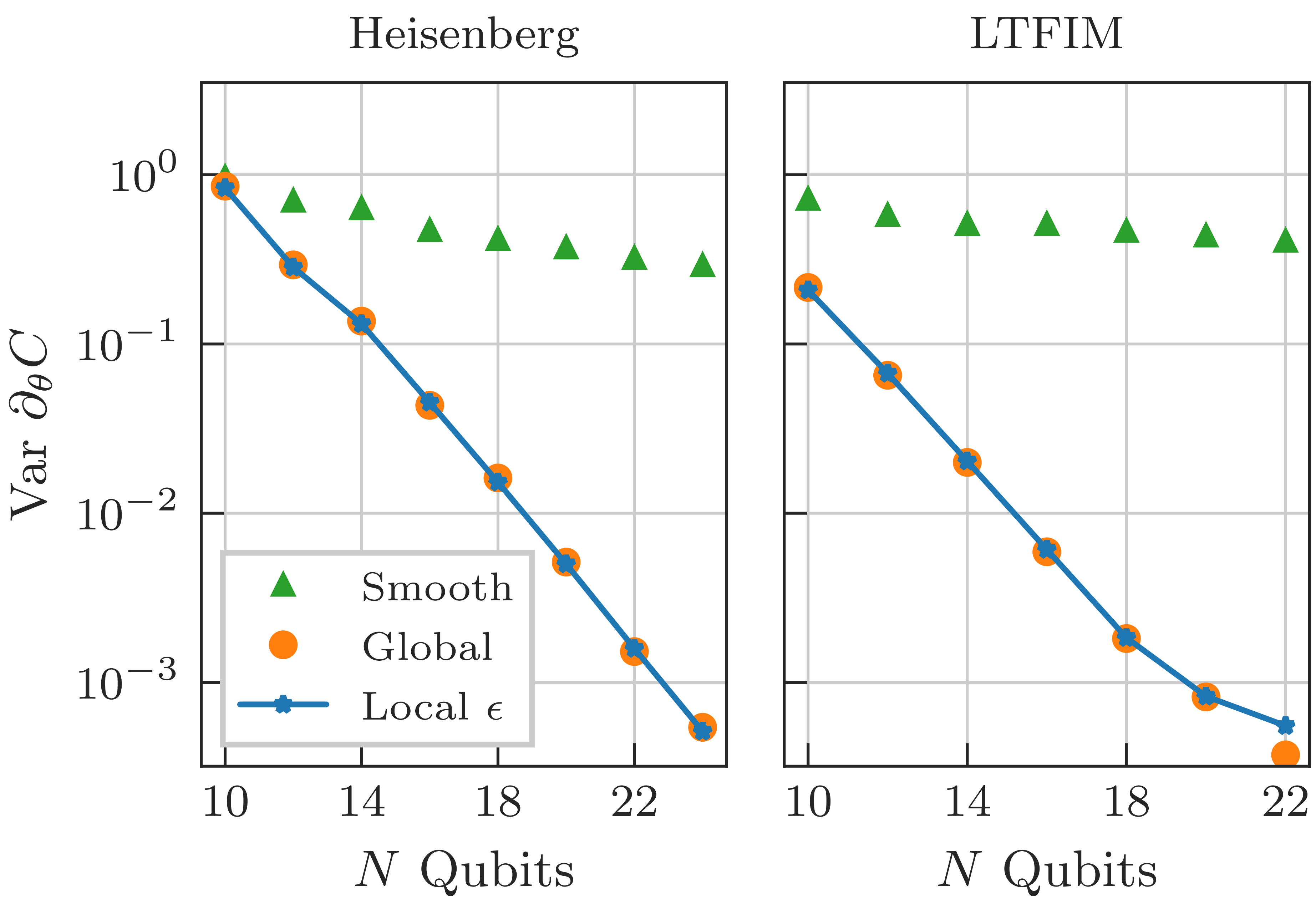
Transferability also for large-N (TFIM)



The warm-start allows to avoid the flat region



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Solution transferability from small to larger system sizes, and from small to larger circuit depths.

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THANKS FOR YOUR ATTENTION!