Learning and testing quantum states of fermionic systems

Antonio Anna Mele
Freie Universität Berlin

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Quantum Physics

[Submitted on 26 Sep 2024]

Optimal trace-distance bounds for free-fermionic states: Testing and improved tomography

Lennart Bittel, Antonio Anna Mele, Jens Eisert, Lorenzo Leone

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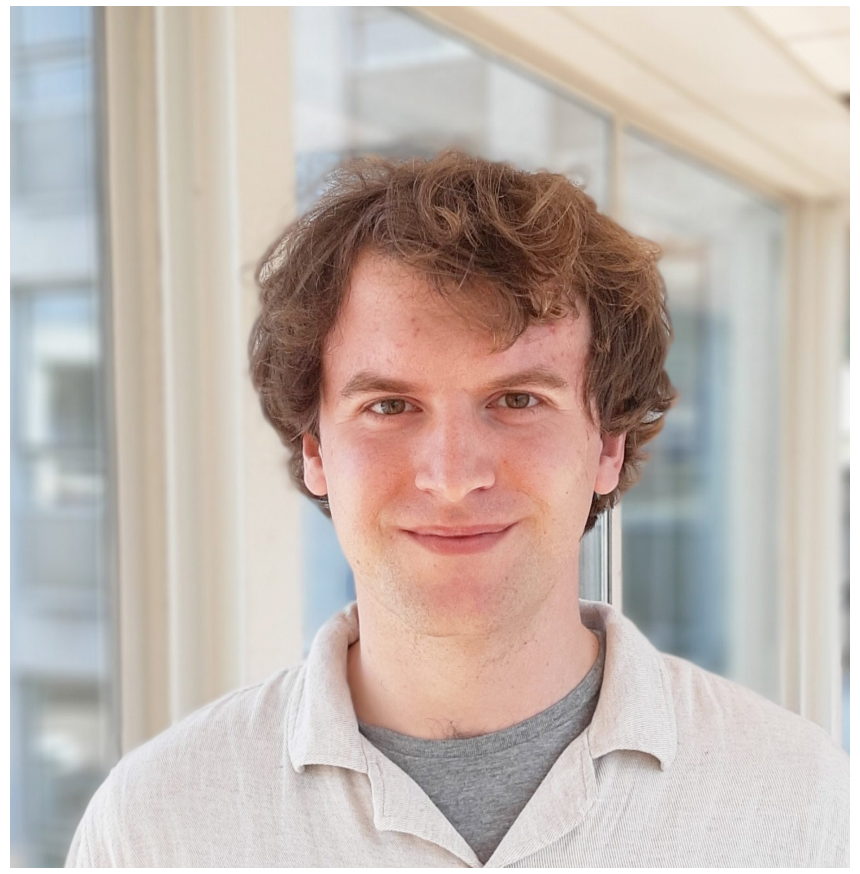
Efficient learning of quantum states prepared with few fermionic non-Gaussian gates

Antonio Anna Mele, Yaroslav Herasymenko

Joint work with great collaborators:

Lennart Bittel	Jens Eisert	Yaroslav Herasymenko	Lorenzo Leone
			
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Outline

- Introduction
- Learning fermionic Gaussian states
- Learning t-doped fermionic Gaussian states

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Introduction

- Advances in quantum technologies have inspired a new field: *Quantum Learning* [1].

- Problem 1: **Learning** quantum states ('tomography').
  - Without any prior assumption, this task is hard. [1] ❌
  - But, if the unknown state belongs to a specific class, efficient learning may be possible. ✔

    (e.g., MPS [2], stabilizers [3], t-doped stabilizer states [4,5], …)

- Problem 2: **Testing** quantum states [6].

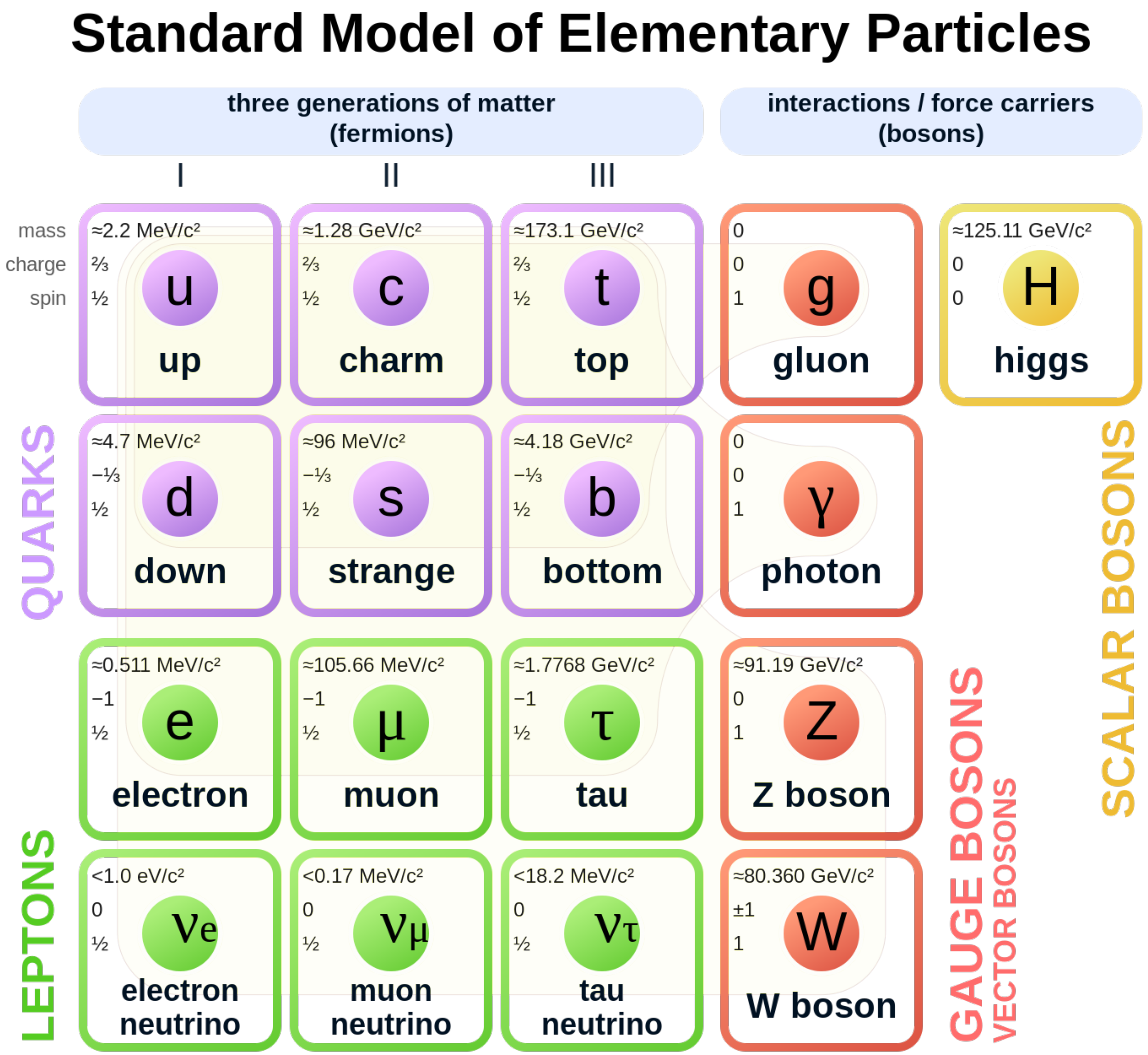
  ("Decide if a state is close to or far from a given class").

  (e.g., Is this state a stabilizer state or not? [7-11])

[1] Anshu et al, A survey on the complexity of learning quantum states, Nature Physics (2024)
[2] Lanyon et al, Efficient tomography of a quantum many-body system, Nature Physics (2017)
[3] Montanaro, Learning stabilizer states by Bell sampling (2017)
[4] Grewal et al, Efficient learning of quantum states prepared with few non-clifford gates (2023)
[5] Leone et al, Learning t-doped stabilizer states, Quantum (2023)
[6] Montanaro et al, A Survey of Quantum Property Testing, Theory of Computing (2013)
[7] Gross et al, Schur-Weyl Duality for the Clifford Group, Comm. in Math. Phys. (2023)
[8] Arunachalam et al, Polynomial-time tolerant testing stabilizer states, (2024)
[9] Hinsche et al, Single-copy stabilizer states, (2024)
[10] Bao et al, Tolerant testing of stabilizer states, (2024)
[11] Liang et al, Tolerant Testing of Stabilizer States with Mixed State Inputs, (2024)

Fermions are ubiquitous in physics

- Fermions are a type of quantum particle. They make up all the matter!



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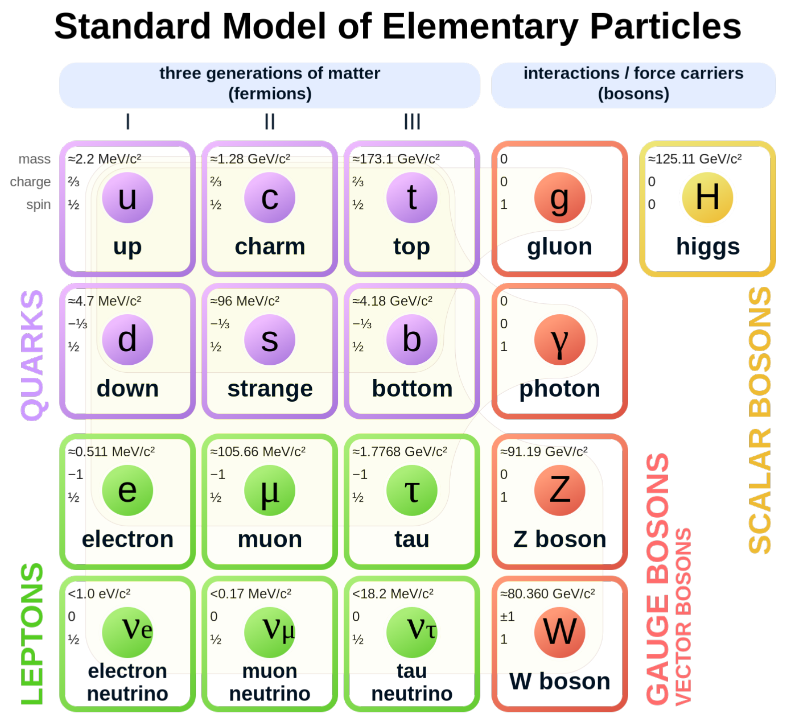
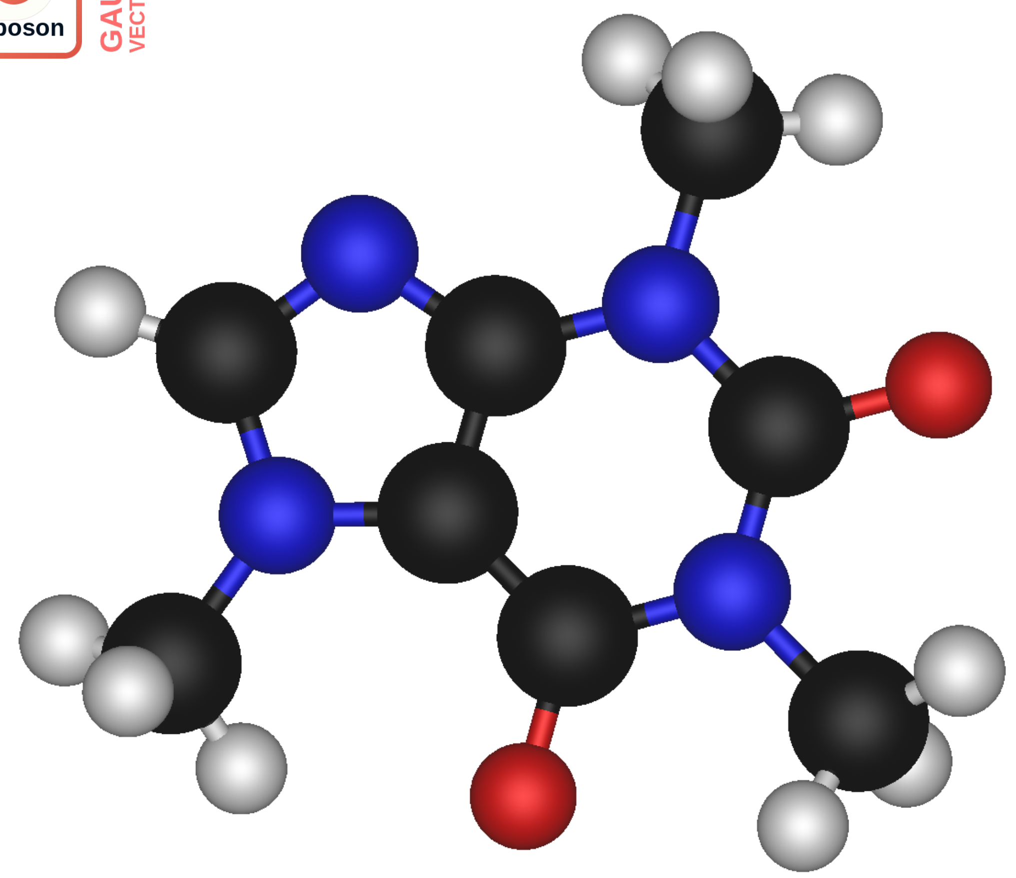
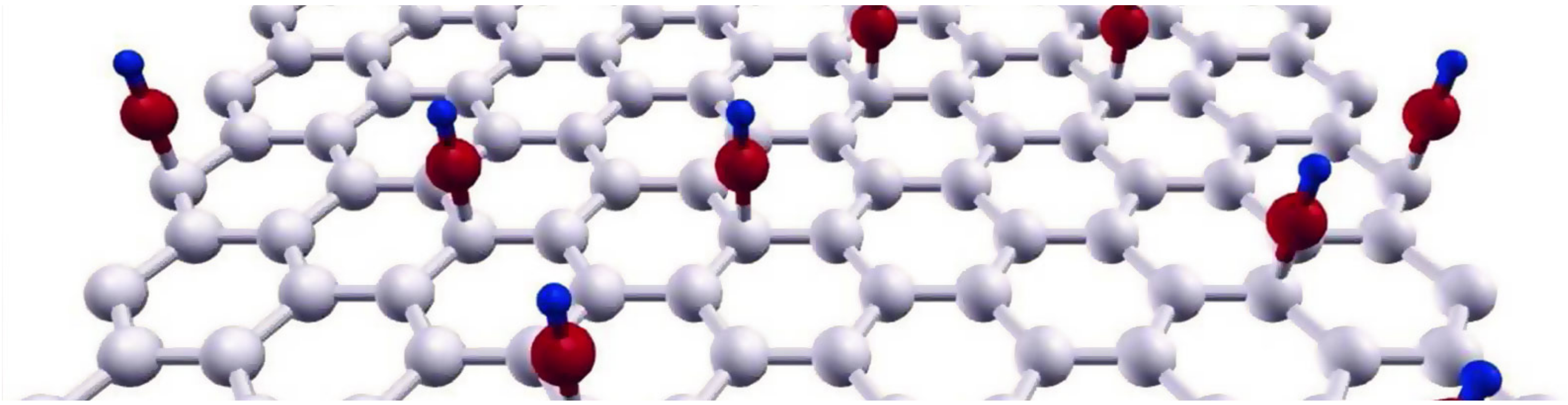


- In all "quantum technologies" (chemistry, semiconductors, etc) of today, fermions—electrons—play a key role.



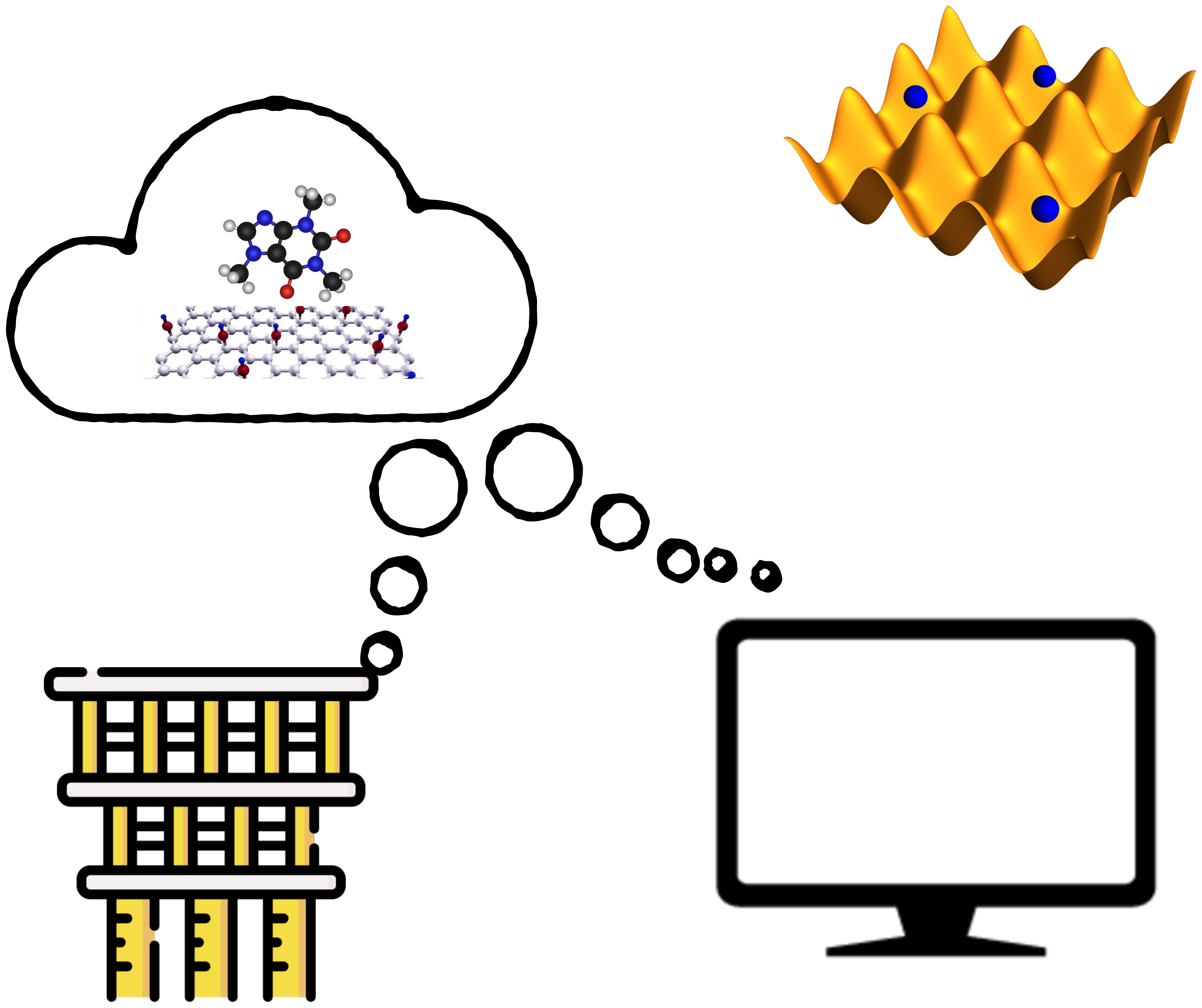


Fermions are ubiquitous in physics

- Fermions are a type of quantum particle.
They make up all the matter!

- In all “quantum technologies” (chemistry, semiconductors, etc) of today, fermions —electrons—play a key role.

- Designing materials and chemicals
= hard computational problems about fermions.



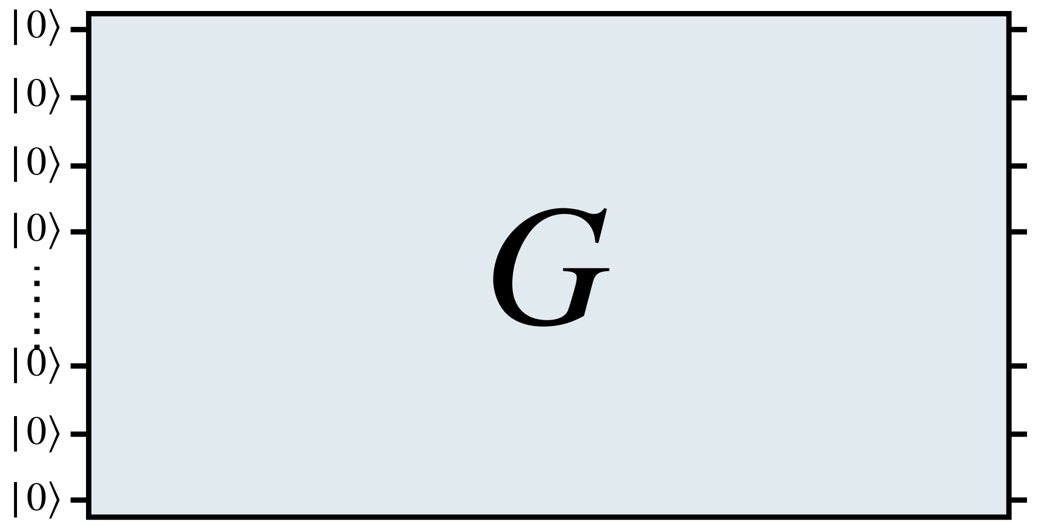
Introduction

- Despite their importance, research on learning fermionic states remains limited.

[11] Aaronson et al, Efficient tomography of non-interacting fermion states (2023)
[12] O'Gorman. Fermionic tomography and learning, (2022), …

<u>Our work</u> aims to provide a comprehensive study on Learning and Testing fermionic states.

✓ We start with the *simplest* fermionic states: 'Gaussian states'.

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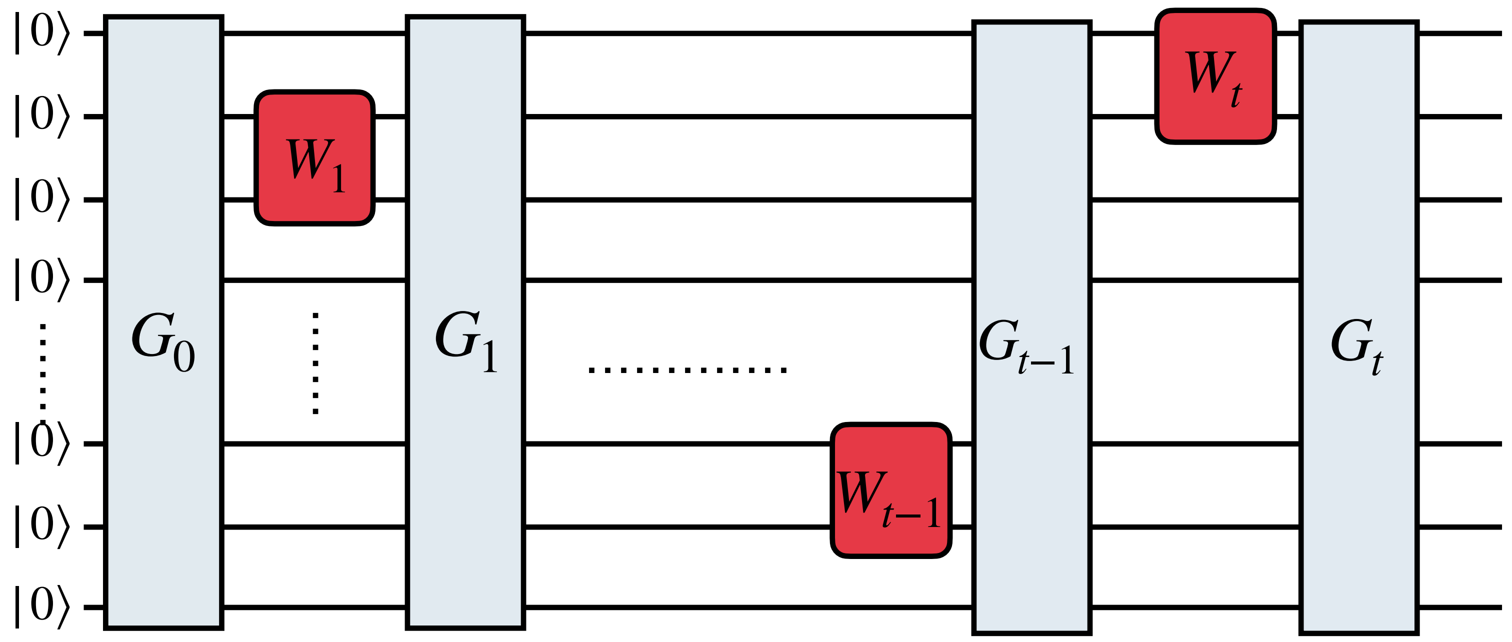
✓ We then analyze more *complex* states: 't-doped Gaussian states'.

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Efficient learning of quantum states prepared with few fermionic non-Gaussian gates

Antonio Anna Mele, Yaroslav Herasymenko

We design *practical* efficient algorithms, while also showing cases where *any* algorithm must be inefficient.

Along the way, we uncover **fundamental properties** of these states.

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Fermionic Gaussian states

(also called free-fermionic states, non-interacting fermions, states prepared by 1D-matchgates circuits, …)

- Fermionic Gaussian states = Gibbs states of “Free-fermions” Hamiltonians

$$\rho = \frac{e^{-\beta H_{\mathrm{free}}}}{\mathrm{Tr}(e^{-\beta H_{\mathrm{free}}})}, \qquad H_{\mathrm{free}} = i \sum_{\mu<\nu\in[2n]} h_{\mu,\nu}\gamma_\mu\gamma_\nu$$

Majorana operators

- Majorana operators:
  (They are just some Pauli strings)

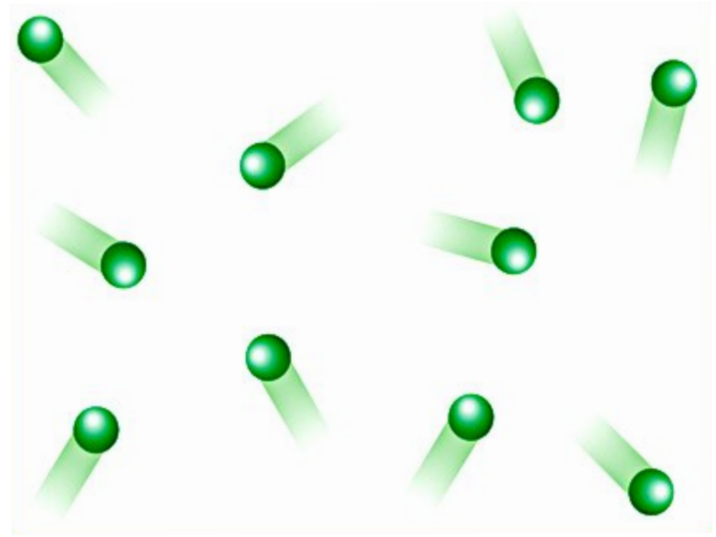
$$\gamma_{2k-1} := \left(\prod_{j=1}^{k-1} Z_j\right) X_k, \quad \gamma_{2k} := \left(\prod_{j=1}^{k-1} Z_j\right) Y_k, \qquad \text{for } k \in \{1,\dots,n\}$$

- Gaussian unitaries: $U = e^{-iH_{\mathrm{free}}}$

- **Why** Gaussian states/unitaries:

  - Model free-fermion physics (many metals, semi- and superconductors)

  - Classically easy to simulate



Fermionic Gaussian states

- Gaussian states ρ are **fully characterized** by their "correlation matrix" $\Gamma(\rho) \in \mathbb{R}^{2n\times 2n}$,

- Correlation matrix $\Gamma(\rho)$ of a quantum state ρ:

$$[\Gamma(\rho)]_{j,k} = -i\,\mathrm{Tr}(\gamma_j\gamma_k\rho),\ \text{for } j<k\in[2n]$$ (anti-symmetric)

How to learn fermionic Gaussian states?

- Gaussian states ρ are **fully identified** by their correlation matrix $\Gamma(\rho)$.
- So it is enough to estimate $\Gamma(\rho)$, but to which accuracy?

Problem (Learning states/Tomography)

Let $\varepsilon > 0$. Given N copies of the (unknown) state $\rho \in \mathcal{S}$, the goal is to output $\tilde{\rho}$ such that (with high probability)

$$\|\rho - \tilde{\rho}\|_1 \leq \varepsilon$$

- We need norm bounds between Gaussian states and their correlation matrices!

(Our first main) Theorem

Let $\rho, \tilde{\rho}$ be Gaussian states, then:

$$\|\rho - \tilde{\rho}\|_1 \leq \frac{1}{2}\|\Gamma(\rho) - \Gamma(\tilde{\rho})\|_1$$

Norm bounds between Gaussian states

Theorem

Let $\rho, \tilde{\rho}$ be Gaussian states, then:

$$\|\Gamma(\rho) - \Gamma(\tilde{\rho})\|_\infty \leq \|\rho - \tilde{\rho}\|_1 \leq \frac{1}{2}\|\Gamma(\rho) - \Gamma(\tilde{\rho})\|_1$$

- "If we know $\Gamma(\rho)$ with accuracy ε, we know the Gaussian state itself with **trace distance** error ε."

Theorem

Let $\rho, \tilde{\rho}$ be pure Gaussian states, then:

$$\|\rho - \tilde{\rho}\|_1 \leq \frac{1}{2}\|\Gamma(\rho) - \Gamma(\tilde{\rho})\|_2$$

- These bounds are **"optimal"** !

How to learn fermionic Gaussian states?

Theorem (Efficient learning of Gaussian states)

$N = O\left(n^{\alpha}/\varepsilon^{2}\right)$ copies of the unknown Gaussian state ρ suffice to learn $\tilde{\rho}$ such that $\|\tilde{\rho} - \rho\|_1 \leq \varepsilon$.

$\alpha = 4$ if ρ is possibly mixed,

$\alpha = 3$ if ρ is pure.

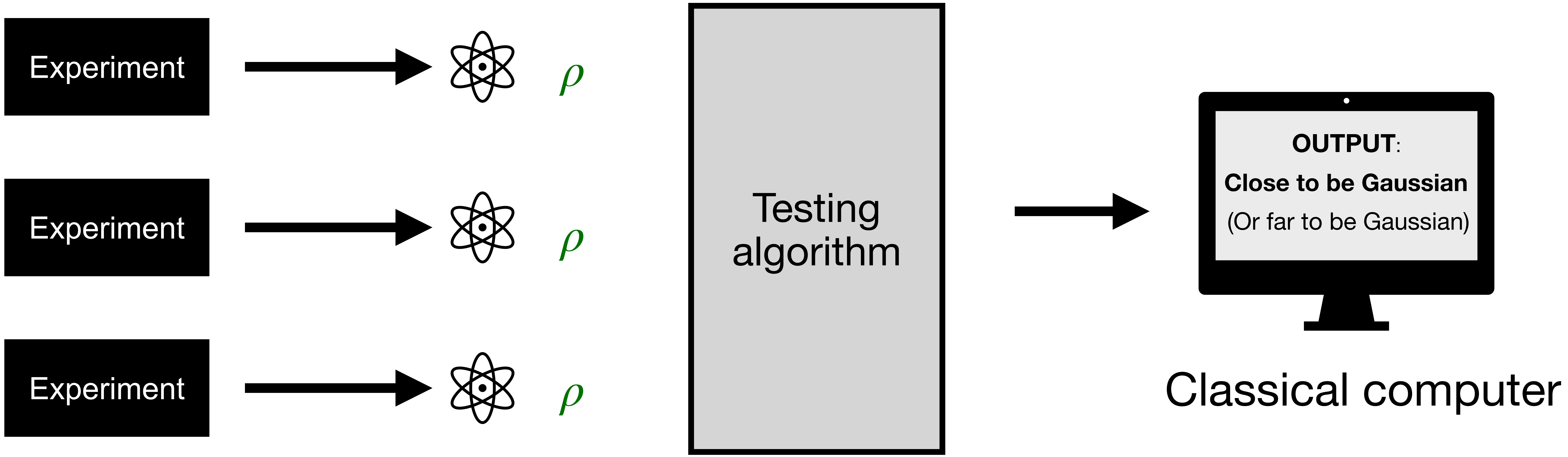
- Previous state-of-art bound (known only for pure-states) was $O(n^5/\varepsilon^4)$, while our is $O(n^3/\varepsilon^2)$.

[11] Aaronson et al, Efficient tomography of non-interacting fermion states (2023)
[12] O'Gorman. Fermionic tomography and learning, (2022)

- The algorithm is just: estimate the correlation matrix and "regularize it".

- **Experimentally feasible protocol:** 'simple' measurements, time-efficient and "noise robust".

Testing whether an unknown state is Gaussian



Problem (Property testing)

Given N copies of the (unknown) state ρ, decide (for $\varepsilon_B > \varepsilon_A \geq 0$) if:

- Case A (ρ is **close** to be Gaussian): There exists a Gaussian state σ such that $||\rho - \sigma||_1 \leq \varepsilon_A$, or
- Case B (ρ is **far** from being Gaussian): $||\rho - \sigma||_1 > \varepsilon_B$, for all σ Gaussian states.

Testing whether an unknown state is Gaussian

Theorem (Testing Gaussian states is Hard!)

To solve the testing problem, $N \geq \Omega(2^n)$ copies of the unknown state are necessary.

$\downarrow$

There is no measure of **'fermionic magic (non-Gaussianity)'** which can be efficiently estimated.

- What if the unknown state—or the states in the Gaussian set—have **rank** $\leq R$?

  $N \geq \Omega(R)$ copies necessary.

- Is there an efficient algorithm for $R = \mathrm{poly}(n)$?

Theorem (Efficient testing for **bounded rank** states)

The Gaussian testing problem can be solved with sample&time complexity $\mathrm{poly}(n, R)$.
(under appropriate conditions on $\varepsilon_A, \varepsilon_B$).

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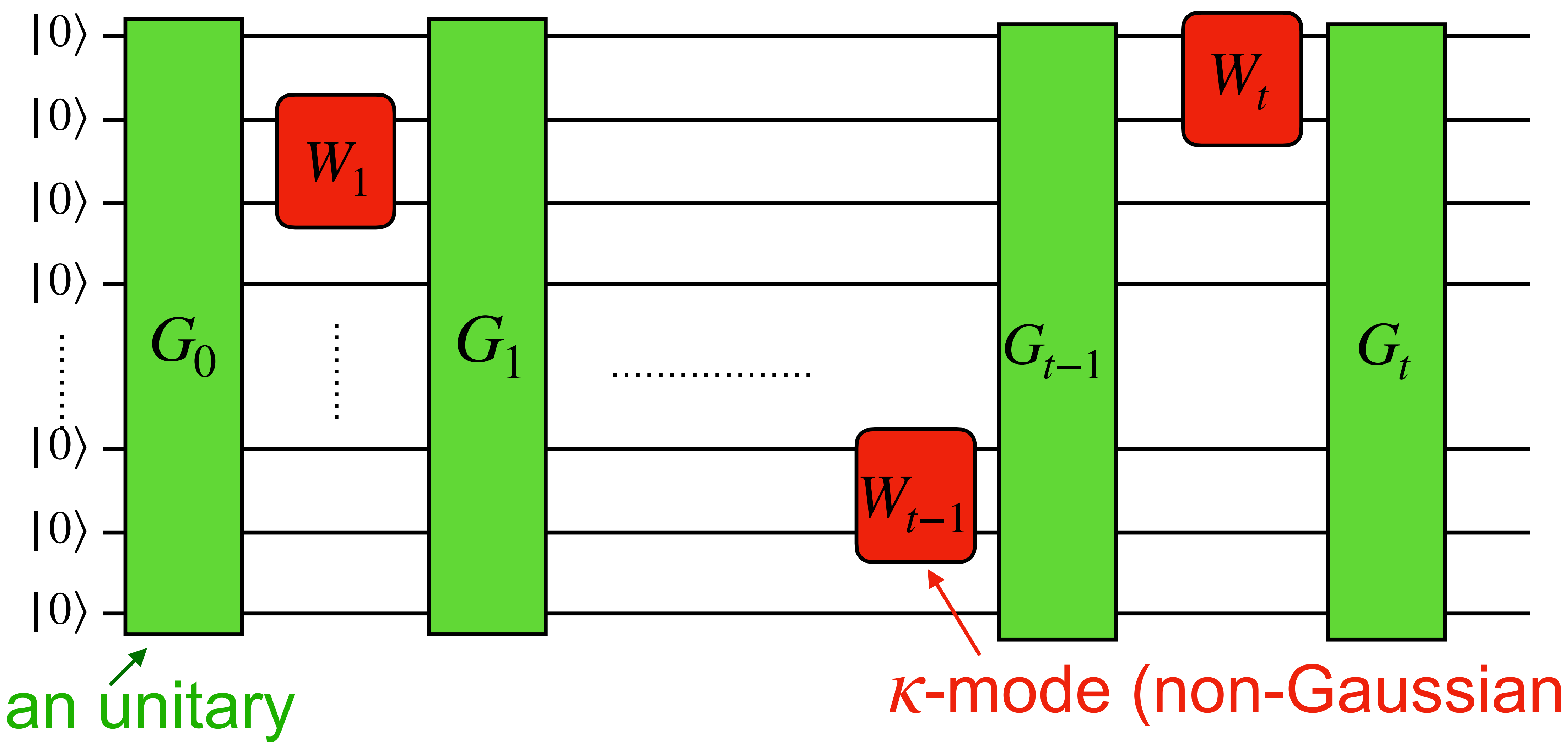
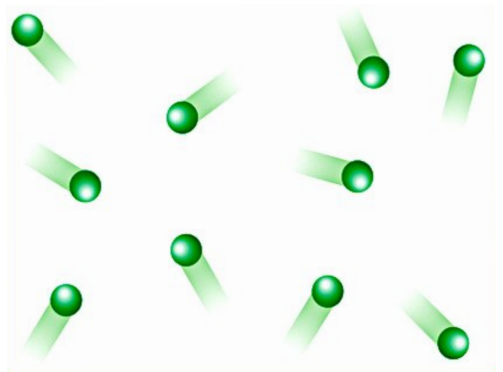
t-doped fermionic Gaussian states

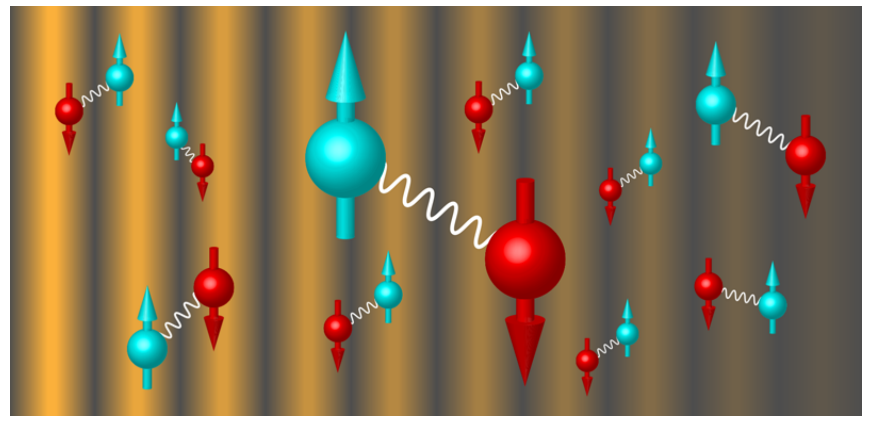
- Gaussian states are efficient to classically simulate and to learn, unlike general quantum states.
- How to interpolate between the two?
- **t-doped Gaussian state** = state prepared by Gaussian (1D-matchgates) unitaries + at most t 'magic' gates.



E.g.:
- $\exp(i\theta\,\gamma_3\gamma_4\gamma_5\gamma_6)$,
- SWAP gate …

- **Why** non-Gaussian circuits/states:
  - They model interacting physics
  - Universal for Quantum Computation

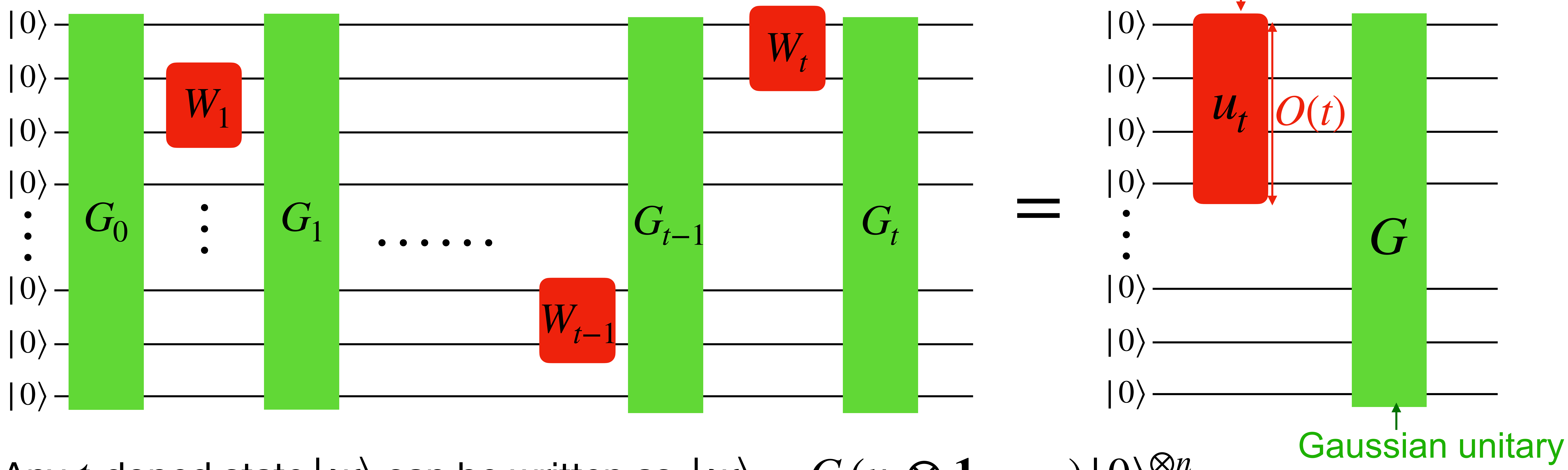




- Classically simulable if $t = O(\log(n))$, no longer for $t \geq \omega(\log(n))$.

What about their **learnability**?
Spoiler: The same! ("New form of complexity")

Theorem (Magic compression theorem):



Any t-doped state $|\psi\rangle$ can be written as $|\psi\rangle = G\,(u_t \otimes \mathbf{1}_{n-O(t)})\,|0\rangle^{\otimes n}$.

Implications:

- More efficient compilation of non-Gaussian circuits ("avoid redundancy").

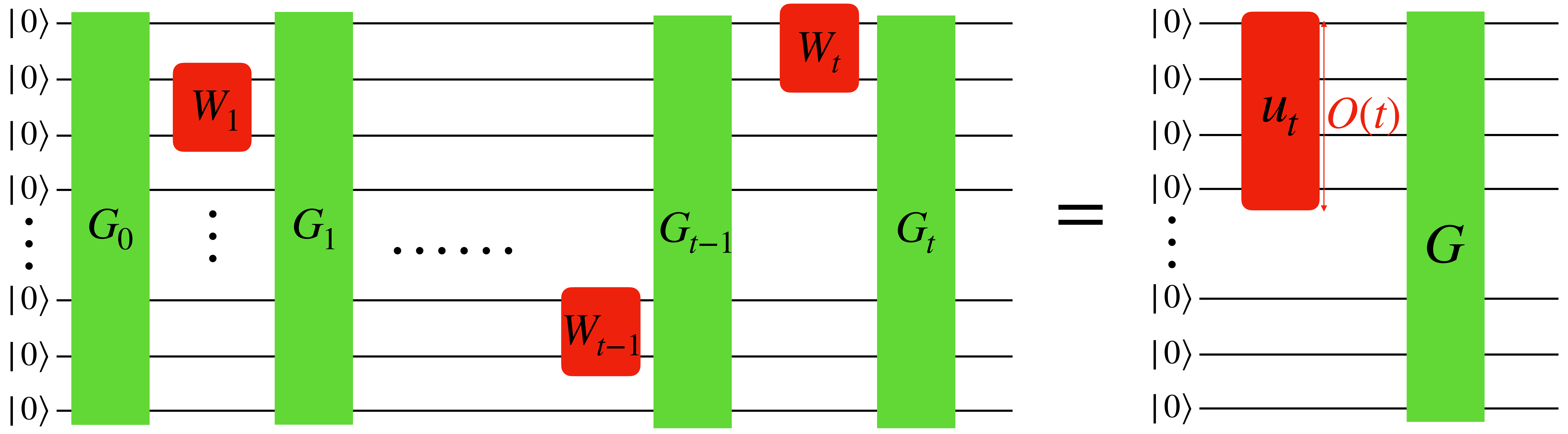
  (Circuit complexity $O(n^2 + t^3)$, compared to the "naive" $O(n^2 t)$)

- Crucial idea for Learning

Analogous theorem holds for "Clifford + T":

[1] Oliviero, Leone, Lloyd, and Hamma, Unscrambling Quantum Information with Clifford Decoders ,Phys. Rev. Lett. 132, 080402 (2024).
[2] Grewal, Iyer, Kretschmer, Liang, Efficient learning of quantum states prepared with few non-clifford gates (2023)

Idea for Learning t-doped Gaussian states



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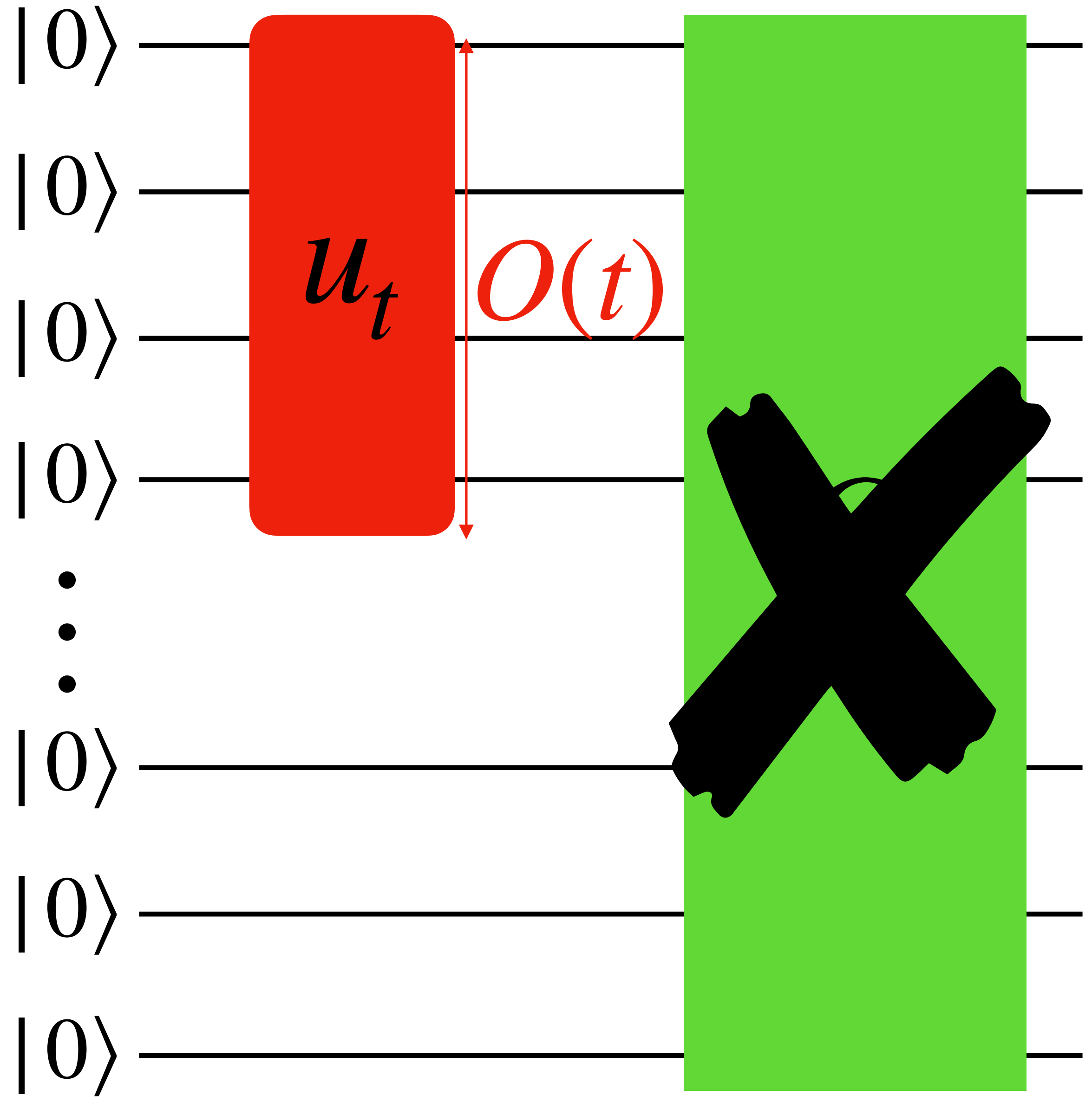
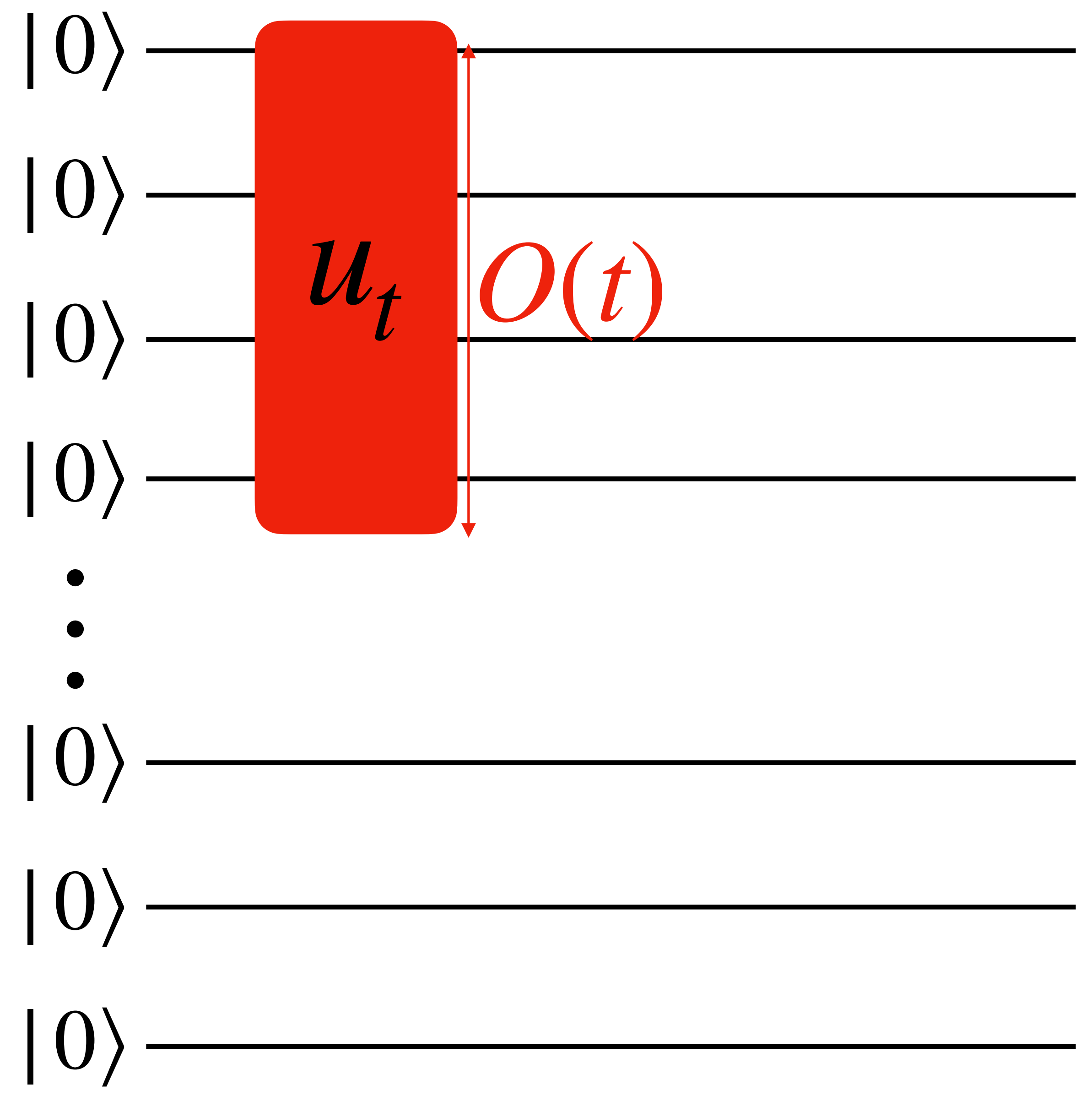


Crucial idea for tomography algorithm:

1) Imagine that we can learn G

2) Apply G^{-1} to $|\psi\rangle$

Idea for Learning t-doped Gaussian states



Crucial idea for tomography algorithm:

1) Imagine that we can learn G (….Yes, we can!)

2) Apply G^{-1} to $|\psi\rangle$

3) Do full state tomography on the first $O(t)$ qubits.

By estimating and processing the correlation matrix of $|\psi\rangle$.

Theorem (Efficient learning of t-doped Gaussian states)

For $t = O(\log(n))$, t-doped Gaussian states can be learnt in $\mathrm{poly}(n)$-time & sample.

- What if t is larger than $\log(n)$?

Theorem (Hardness learning of $\omega(\log(n))$-doped Gaussian states)

If $t \geq \omega(\log(n))$, there is no $\mathrm{poly}(n)$-time algorithm to learn t-doped Gaussian states, up to common crypto-assumptions (i.e., "RING-LWE cannot be solved by quantum computer in sub-exp-time").

- The runtime of our algorithm $\mathrm{poly}(n, 2^t)$ is "optimal".

Further remarks

- **Experimentally feasible protocol:** single copy, "simple" measurements, "noise robust".

  ("approximate t-doped"/mixed state learning).

- Our algorithm extends to all "t-compressible states". (e.g., ground states of impurity models [1])

$$|\psi\rangle = G\,(u_t \otimes \mathbf{1}_{n-O(t)})\,|0\rangle^{\otimes n}$$

- We provide an efficient testing algorithm for t-compressible states.

[1] S. Bravyi and D. Gosset, Complexity of quantum impurity problems, Commun. Math. Phys. 356, 451–500 (2017)

Summary

- Optimal trace distance bounds for Gaussian states, and efficient learning.
- Hardness for testing general Gaussian states, but efficient for low-rank states.
- Magic-compression theorem for t-doped states, and efficient learning/testing of t-compressible.
- Critical threshold for efficient 'Learnability' $= \log(n)$ magic gates.

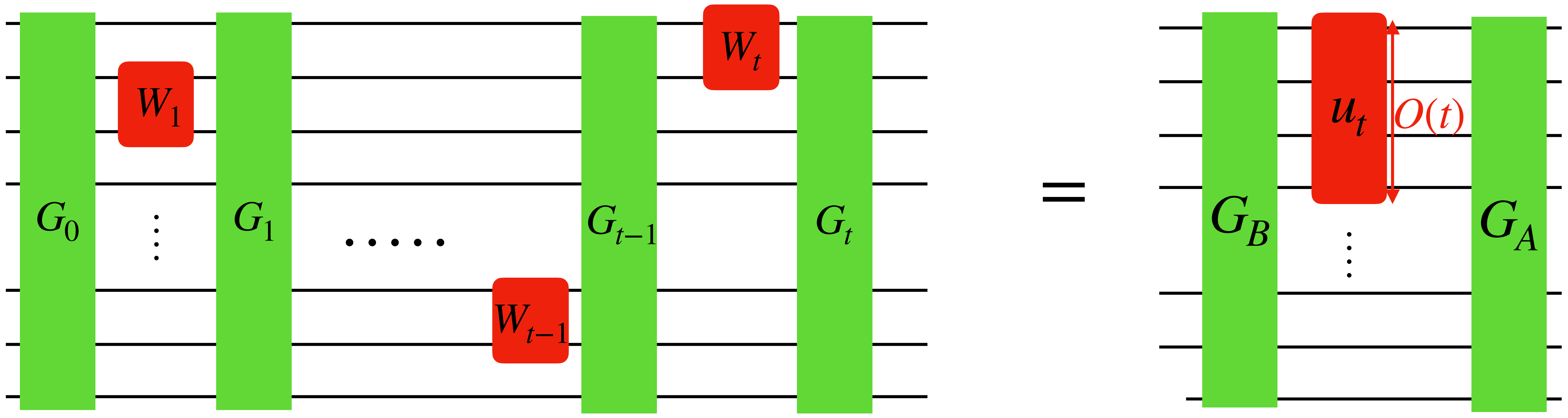
$\downarrow$

"A new form of state-complexity coming into play".

Open questions

- Learning t-doped Gaussian unitaries. (They can be 'compressed' as well, i.e., $U_t = G_A(u_t \otimes I_{n-O(t)})G_B$)

($t = 0$ already solved [1].)



[1] M. Oszmaniec, N. Dangniam, M. E. Morales, and Z. Zimboras, Fermion sampling: A robust quantum computational advantage scheme using fermionic linear optics and magic input states, PRX Quantum, 2022.

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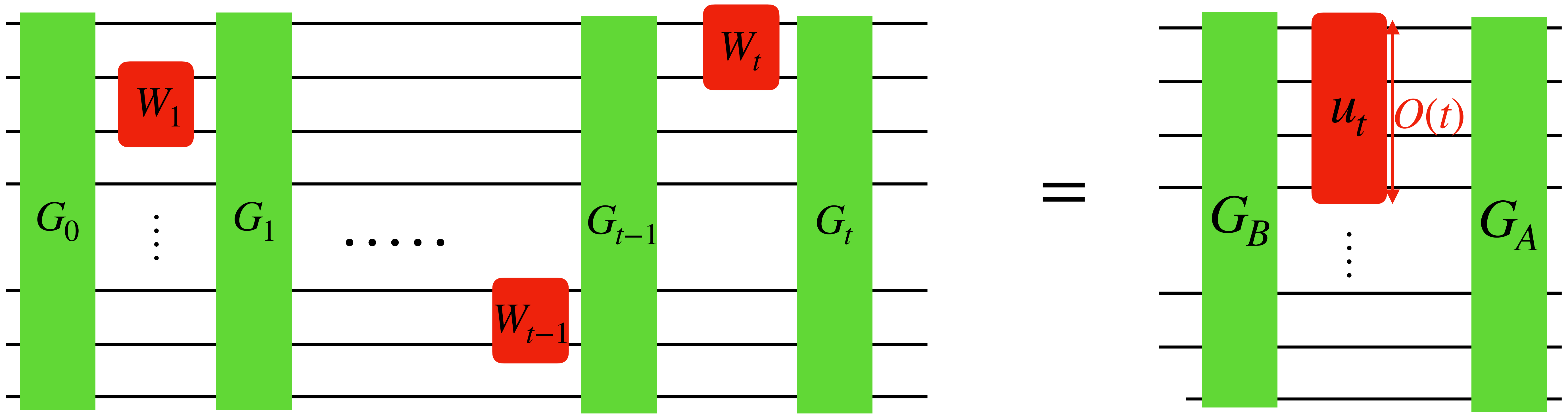
- Agnostic tomography.
- Testing Gaussian unitaries.
- Optimal learning of free fermions.



Thank you for your attention!

[1] M. Oszmaniec, N. Dangniam, M. E. Morales, and Z. Zimboras, Fermion sampling: A robust quantum computational advantage scheme using fermionic linear optics and magic input states, PRX Quantum, 2022.